
Bilateral localization algorithm for wireless sensor networks with communication holes

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Abstract: Localization is one of the vital services for many protocols and applications in wireless sensor networks. Although in most of the range base localisation algorithms an unknown node requires at least three anchor nodes to localize itself, some bilateration approaches can perform the task with only two anchors. These approaches lose their accuracy in cases when there is sparsity of nodes at some spot in the field. A communication hole is a problem that localization algorithms face, causing their performance reduction. In this paper, we devise a new recursive approach, called Non-directed Bilateration Position Estimation (NBPE) that uses only two anchor nodes to calculate two possible positions and prune one of them using a specific criterion. To decrease error propagation a precedence mechanism is applied to select the proper anchors, which results in higher accuracy in networks with communication holes. NBPE is evaluated through simulation under various noise conditions and network densities.

Keywords: Node localization; Prune base algorithm; NBPE algorithm; Holes problem; Wireless sensor networks.

1 Introduction

In many wireless sensor network applications it is vital for sensor nodes to be aware of their location in order to be able to accurately analyze the collected information originated from source nodes. Without knowing the physical location of the nodes, collected data by sensor nodes may not be useful enough (Malazi & Zamanifar 2010, Malazi et al. 2011). The location information also can be helpful in geographical-based routing algorithms which require the position of a destination node (Li et al. 2000). There exist several cases in which sending extra packets can be avoided by location awareness of neighboring and destination nodes leading to energy saving (Yu et al. 2001). Another application of location awareness is in density control algorithms where node

are gridded and state indication is performed based on position information (Jiang & Dou 2004). Finally, location awareness of nodes is needed for maintenance activities (Dargie & Poellabauer 2010).

In addition to locate nodes physically in relative or absolute coordination, localization algorithms should satisfy specific restrictions of WSNs such as memory and energy shortage, fault tolerance, dynamic topology, scalability, processing limitation, and low network development cost (Dargie & Poellabauer 2010, Srinivasan & Wu 2007, Wang et al. 2010, Bulusu et al. 2000). Therefore, a suitable algorithm is the one that performs well in different network density with low localization error and energy consumption, and can locate more localizable nodes.

Global positioning system (GPS) is one of the conventional locating systems in different applications and networks. But using the system in wireless sensor networks faces several obstacles: First, GPS receivers are highly energy consuming devices. Second, equipping

each node with GPS receivers will increase the price of the node, resulting an increase in network development price. Additionally, it is not possible to use GPS in indoor applications and cloudy climates. These factors put limitations to equipping nodes with GPS receivers.

Localization methods can be divided into two major central and distributed categories (Wang et al. 2010, Kulaib et al. 2011, Das et al. 2011). In central ones, nodes measurement and neighbouring information are sent to a processing centre, where network graph information is stored, and the locations are calculated with mathematical rules and formulas. Accordingly, the results are sent to the nodes. High communication and time costs are the main drawbacks of this category, resulting in low scalability and high energy consumption.

In distributed approaches, the nodes themselves have the localization duty. These methods apply a kind of cooperation and distributed calculation among the nodes (Basagni et al. 2004). Generally, the method imposes low communication cost ending low energy and time consumption in comparison with central ones. Consequently, they are more suitable for WSNs compared to their central rivals. Most of the localization algorithms are placed in this category, however, it can be claimed that none of them is able to fulfil WSN requirements completely (Sohraby et al. 2007, Rahman & Kleeman 2009). Some of them do not use anchor nodes, which leads to high energy consumption and response time (Iyengar & Sikdar 2003, Čapkun et al. 2002, Zhu et al. 2010), some need special requirements causing higher node price (Kim et al. 2009), some have good performance in small networks, while having problem in larger ones (Pugh & Martinoli 2006), some having merely good performance just for special geography (Hu et al. 2008a, Albowicz et al. 2001) and some others doing recursive localization that use fewer anchors increases localization error through the network (Albowicz et al. 2001). The hole existence in the network is another problem facing most of the distributed methods (Yang & Liu 2012, Liu et al. 2010) that cause either to increase localization error or to decrease the number of localizable nodes ending in performance loss (Aspnes et al. 2006).

Since nodes are distributed randomly there might exist communication holes at the beginning of dispensation. These holes can make localizable nodes decrease and also in some bilateral algorithms such as DPE (de Oliveira et al. 2009) causes wrong position estimation of some nodes.

In this paper, we propose a distributed and recursive approach named Non-directed Bilateral Position Estimation (NBPE) that locates the nodes by using the precedence mechanism to anchor nodes and selecting a proper pair of them having at least one common anchor node. The approach controls the irregular error propagation through the network. It also has low energy consumption and response time due to simple computation and lower communication complexity. NBPE utilizes the least anchor nodes to unknown nodes

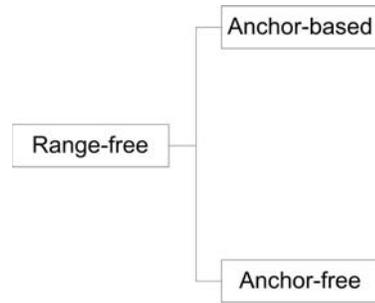


Figure 1: Range-free approaches categorization.

localization, easily escapes communication holes so that holes existence has lower defect on localized nodes number and positioning error. Moreover, it is scalable in terms of network size. Since each node uses the least requirements (two anchor nodes) to locate itself, NBPE stands against increasing number of not localized nodes and error propagation, while encountering holes.

The remainder of the paper is organized as follows. Section 2 provides state-of-the-art on WSN localization algorithms. Problem definition is explained in detail in Section 3 and in Section 4 the proposed approach, NBPE, is described. In Section 5 the evaluation is presented and finally, the conclusion comes in Section 6.

2 Related works

Localization algorithms consist of two major phases (Basagni et al. 2004, Yang & Liu 2012, de Oliveira et al. 2009). In the first phase, named *measurement*, the nodes do their estimated measurements using the received signal features and prepared neighbouring information. In the second phase, called *combination*, the nodes are located using the information collected in the first phase. The main task of a localization algorithm is to use the information from the first phase and to derive the location of nodes in the network accurately. The localization accuracy depends on the approach used in the second phase as well as the accuracy of measurements in the first phase. As it is formerly said, distributed approaches are suitable for WSNs. The approaches are classified into two major groups of range-based and range-free algorithms.

2.1 Range-free approaches

This class of algorithms uses neighbouring information and its radio coverage to calculate the location of nodes. The main advantages of these approaches are simplicity and rapid localization (Gui et al. 2014). These approaches are further divided into anchor-based and anchor-free approaches.

In anchor-based approaches locations of *unknown* nodes are computed using the location information of anchor nodes. Centroid (Bulusu et al. 2000) is one of these algorithms that performs according to

neighbouring information of surrounding anchor nodes. The locations of unknown nodes are calculated as an average of neighbouring anchor nodes. The accuracy of the estimation depends highly on the number of anchor nodes. In order to solve this issue in (Bulusu et al. 2000, Sundani et al. 2011), Centroid is implemented recursively and weight is considered for anchor nodes, but in these approaches localization error increases impressively through the network. APIT (He et al. 2003) is another approach in which every *unknown* node performs *Point In Triangulation* (PIT) test. The test determines triangle zones belonging to them. Afterwards, it localizes itself as the centre of common triangle zones placed in. Similar to Centroid, in order to achieve higher accuracy, high density of anchor nodes is required. To oppose this problem Gradient (Nagpal et al. 2003) and DV-Hop (Niculescu & Nath 2003b) are proposed. In Gradient (Nagpal et al. 2003) unknown nodes apply hop number multiplication by node's radio coverage to measure the distance from anchor nodes, but in DV-Hop (Niculescu & Nath 2003b) average hop distance is estimated considering the distance between two anchor nodes and the hop count. The distance between each unknown node and anchor nodes is estimated by hop count multiplied by the average hop distance. Whenever the distances to at least three anchor nodes are approximated, both methods apply multilateration to estimate nodes location.

In anchor-free approaches, none of the nodes are equipped with additional devices, such as GPS receivers. Hence, none of them are aware of their locations, and positions are estimated relatively. MDS-MAP (Niculescu & Nath 2003b) is one of the algorithms in this category which uses a two-step approach. In the first step, distance matrix between all pair of nodes is calculated based on the hop count multiplied by the radio range. In the second step, local coordination is formed using formation and analysing vectors. The approach complexity is from $O(n^3)$. This approach can also be implemented using a few anchor nodes, therefore a third step consists of flip, rotation and scaling operations is required to be added.

Most of range free algorithms suffer from poor localization accuracy.

2.2 Range-based approaches

In this category of algorithms, node location is calculated according to the received signal features and neighbouring information. The localization accuracy is mainly dependent to the transceiver's accuracy and environmental noise. Some of the commonly used features of the communication signal are as follows. Figure 2 categorizes range-based approaches based on (Srinivasan & Wu 2007, Wang et al. 2010, Kulaib et al. 2011, Raghavendra et al. 2004).

Angle of Arrival (AoA): Anchor nodes making thin and rotating beams are used. The other nodes are aware of the speed of signal propagation and rotation.

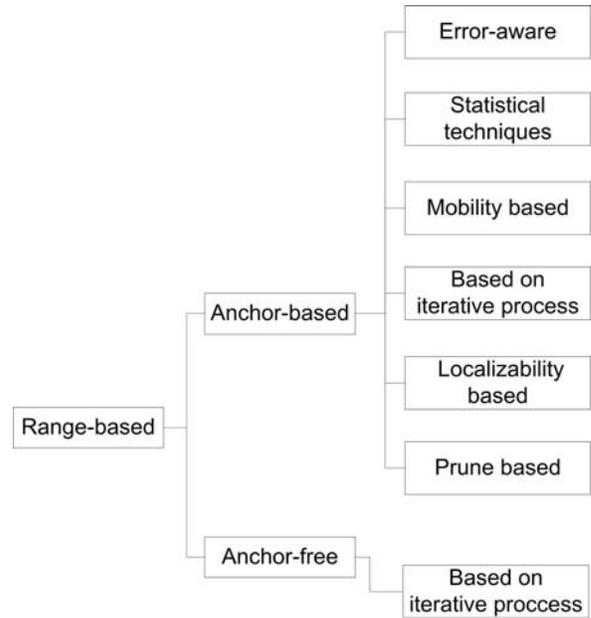


Figure 2: Range-based approaches categorization.

Triangulation technique is used to estimate the nodes positions (Iyengar & Sikdar 2003, Niculescu & Nath 2003a, Benbadis et al. 2005, Hu et al. 2008b). Directional antennas are required to measure this feature.

Received Signal Strength (RSS): RSS indicator is used to measure this feature existing on node's radio transceivers and this prevents extra cost of network development. The distance between two nodes is estimated by Friss (Rappaport et al. 1996) attenuation signal model having received and sent signal power. Equation 1 demonstrates the model, where P_r is the received signal strength (i.e., power), P_t and G_t are the transmitted power and antenna gain, respectively. G_r is the receiver antenna gain, d is the distance between the receiver and transmitter antennas, L is the system loss factor ($L \geq 1$), and λ is the wavelength in meters.

$$P_r = \frac{P_t \cdot G_t \cdot G_r \lambda^2}{(4\pi)^2 d^2 L} \quad (1)$$

Common environmental noise and signal propagation models must be determined prior to apply this feature. RADAR (Bahl & Padmanabhan 2000) is one of the first approaches using RSS to estimate the distances. Researchers in (Rahman & Kleeman 2009, Albowicz et al. 2001, Yang & Liu 2012, de Oliveira et al. 2009, Alsindi et al. 2006, Huang et al. n.d.) introduces approaches based on this feature.

Time of Arrival (ToA): The feature indicates signal fly time from source to destination. That is, the receiver sends signal instantly after receiving it and the sender calculates ToA by subtracting receiver sending delay from the whole time and by making it in half. Pelusi (Pelusi et al. 2006) and Borgia (Borgia et al. 2005) use the feature in their works.

Time Difference of Arrival (TDoA): To measure this feature, the sender must have the capability of sending two different types of signals. Usually one is radio and the other one is ultrasound. The receiver estimates the distance by the difference between two signal arrival times. The feature requires additional devices which leads to higher network development costs. In (Priyantha et al. 2000) TDoA is used to estimate the distance.

2.2.1 Anchor-based approaches

In this class of algorithms position information of anchor nodes are used to estimate the location of other nodes. Figure 2 depicts various classes of this category.

Mobility based: This class of algorithms benefits from one or more mobile anchor nodes. The anchor nodes received their location via GPS receivers and broadcast it periodically. The rest of the nodes are able to estimate their location by at least three available positions. The research in (Kim et al. 2009), suggests a spiral anchor node movement. Hul et al. in (Hu et al. 2008b) uses relay nodes to send anchor node positional information. The main criterion to choose the relay node is how long it stays neighbouring the anchor node during movement.

Error aware: Combination of error features with localization algorithms are considered to enhance algorithms against environmental noises. Distance measurement, and location estimation error are two kinds of error propagated through the network. Authors in (Zheng et al. 2014) introduce a probability factor to show the credibility of distance measurement, then the factor is used in optimizing the performance index. In (Kim et al. 2013) Cramr-Rao Bound is used as a criterion for selecting an appropriate access point set to localize sensor nodes. PML (Rahman & Kleeman 2009) is another algorithm applies the priority mechanism to anchor nodes in order to decrease the influence of environmental noise. It selects pairs of anchors being approximately in the same distance from the unknown node. Finally, after selecting two anchor pair, unknown node location is estimated by two hyperbolic intersections.

Statistical techniques: These approaches are preferred in noisy environments and non-deterministic measurements. Bayesian filter derivatives including Kalman filter (Welch & Bishop 1995) and Particle filter (Arulampalam et al. 2002) are used for statistical location estimation through noisy measurements. LaSLAT (Taylor et al. 2006) is a framework used to locate mobile nodes based on a Bayesian filter updating location estimation using new measurements periodically. In Mont Carlo Localization (Langendoen & Reijers 2004) Unscented Kalman filter injected to produce estimated positions. A method according to probable RSS model is proposed in (Peng & Sichitiu 2007), which primarily unknown nodes guess their position, then they update their position by receiving

packets from neighbouring nodes and using distance probability density functions.

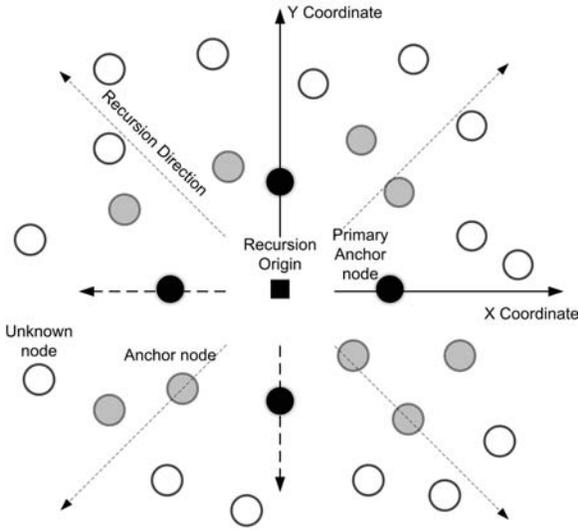
Based on iterative process: In these approaches iterative multidimensional scaling (MDS) runs over local range measurements to form local maps. The route from one anchor to another adjusts local maps. Absolute location is gained by stitching local maps where at least three anchor nodes in neighbouring local maps are needed (Kwon & Song 2008). dwMDS (Sheng & Garimella 2012) begins with a guess of location and makes away using an iterative manner. This algorithm causes improvements in MDS, so that processing complexity is decreased from $O(n^3)$ to $O(n.L)$ where L illustrates the number of iterations for accuracy level satisfaction. There is another approach in (Sheng & Garimella 2012) to enhance MDS in absolute mapping process that uses a sensor fusion approach to estimate nodes location.

Localizability based: The approaches consist of two main phases. In the first, nodes' localizability are checked and in the second, location of localizable nodes are estimated. (Yang & Liu 2012, Jackson & Jordán 2005) are instances discussing the rigidity graphs and their usage in localizability. RMBE (Wang et al. 2011) determines the total realization of the network sub graph in Euclidean space and then prune invalid realizations to remain just one valid realization. Wheel graphs having trilateration graphs as a subset in (Yang et al. 2009) is used for checking localizability. It proves, if a node belongs to this graph, there is absolutely a unique realization in Euclidean space. Time complexity of this graph in complete forms are $O(n^3)$ and in sparse ones are $O(n)$.

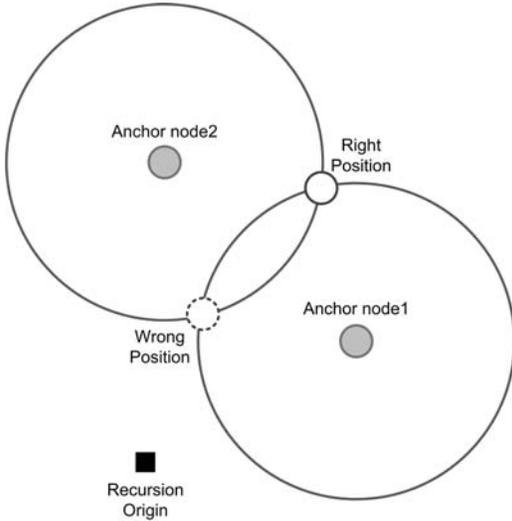
Prune based: This set of approaches determine probable positions for a node by applying geometrical rules, then invalid positions are pruned. Hence, the valid one remains. They usually lead to lower use of anchor nodes. Directed Position Estimation (DPE) (de Oliveira et al. 2009) calculates two possible positions for an unknown node using the two neighbouring anchor nodes and prune one of them. The approach starts from network centre with four primary anchor nodes and proceeds recursively in four directions (Figure 3a). The criterion to prune the wrong position is nearness to recursive origin (Figure 3b). Recursive origin's coordinate is the centre of four primary anchor nodes. As Figure 4 shows the approach is composed of four steps. First, an unknown node selects two neighbouring anchor nodes. Second, it measures the distance from itself to them. Third, two probable positions are calculated and the farther one from the recursion origin is selected. Finally, the unknown node turns into an anchor node and broadcasts its position.

2.2.2 Anchor-free approaches

Basically, these approaches are based on iterative process, and composed of 3 phases. First, the network is divided into sub networks connected together by one



(a) Square structure of primary anchor nodes, proceeds localization in four specified direction.



(b) Probable position farther from recursion origin is selected as a valid one.

Figure 3: Pruning in Directed Position Estimation algorithm (de Oliveira et al. 2009).

or more sensors nodes named coupler nodes (common in two sub networks). Second, the nodes position is calculated local coordination systems and third, every local coordination is converted to global coordination by coupler nodes and flip, reflection and rotation operations. Several researches follow this approach (Zhu et al. 2010, Benbadis et al. 2005). Authors in (Zhu et al. 2010) apply clustering to solve the scalability problem.

2.3 Recursive and non-recursive approaches

The approaches mentioned so far can be implemented in recursive and non-recursive forms. In recursive approaches any unknown node turns into an anchor node

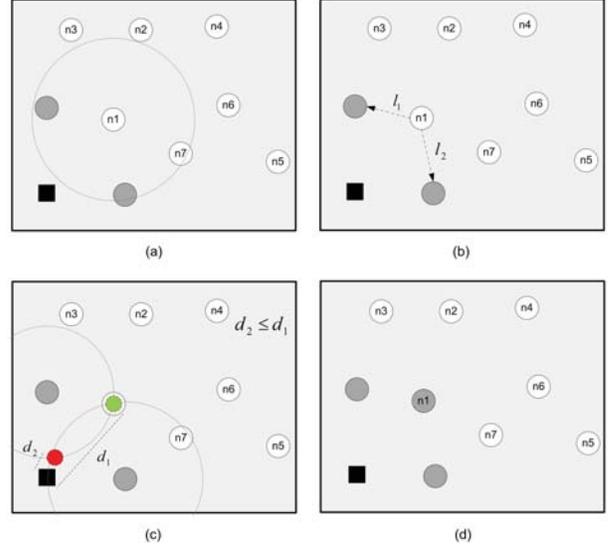


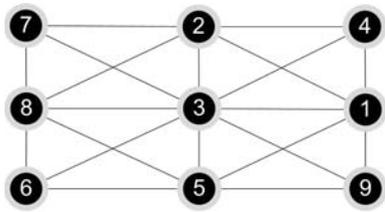
Figure 4: Main localization steps in DPE. (a) The unknown node selects two neighbouring anchor nodes. (b) Measure its distance from itself to them. (c) Two probable positions are calculated and the farther one from the recursion origin is selected. (d) Turns into an anchor node and broadcasts its position.

after localization and broadcasts its position whereas, in non-recursive way the last operation that each unknown node does, is to estimate its position. RPE (Albawicz et al. 2001) is a recursive algorithm that every unknown node applies trilateration technique using three neighbouring anchor nodes to estimate its own position, then it turns into an anchor node and finally, broadcasts its position.

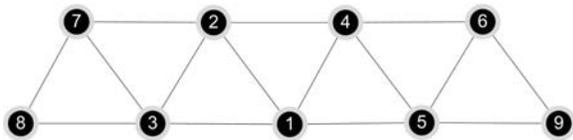
Generally, it is concluded from the surveyed approaches that distributed ones are more compatible with WSNs for scalability and lower energy consumption reasons. Among the approaches, range-based algorithms have priority over range-free ones because of higher accuracy, in the absence of extra range measurement devices. Moreover, anchor nodes in range-based localization make the approaches more convenient. Anchor-based algorithms will be more compatible if they use limited number of nodes equipped with GPS receivers otherwise, they will impose extra expenses for the network development. On the other hand, one of the biggest inadequacy of recursive approaches is propagating irregular localization error though the network. The inadequacy can be improved by propagated error controlling

3 Problem Definition

It is assumed that n nodes are located in distinct physical positions across a two dimensional Euclidean geometry sensor field. The network is considered as a weighted graph $G = (V, E)$ where $V = \{v_1, v_2, \dots, v_n\}$ represents



(a) Trilateration ordering.



(b) Bilateralation ordering.

Figure 5: Ordering in different localization methods

the set of sensor nodes; $e_{ij} \in E$ iff distance between v_i , v_j is less than t_r (maximum transmission range node v_i and v_j). w is defined as $w_{ij} \leq t_r$ is the weight of edge e_{ij} that implies the distance between nodes v_i and v_j .

One type of efficiently localizable networks is trilateration network. A graph has a trilateration ordering with seeds v_1 , v_2 and v_3 if its vertices can be ordered as $v_1, v_2, v_3, \dots, v_n$ so that v_1, v_2 and v_3 induce a complete sub-graph, and each v_i ($i > 3$), is adjacent to at least three vertices v_j where $j < i$. A network is called trilateration network, if its graph has a trilateration ordering (Figure 5a).

A graph has a bilateralation ordering with seeds v_1, v_2 , and v_3 if its vertices can be ordered as v_1 to v_n so that v_1 and v_2 are adjacent, and each v_i ($i > 2$), is adjacent to at least two vertices v_j where $j < i$. Graphs with bilateralation orderings are called bilateralation graphs, and a network is named a bilateralation network if its graph has a bilateralation ordering (Figure 5b).

Prune-based approaches are a subset of multi-point approaches in the graph theory. A multi-point $P = \{p_1, \dots, p_n\}$ in d -dimensional space is a set of n points in R_d labelled p_1, \dots, p_n . A multi-point P is generic if the coordinates of points in P are algebraically independent over the rational. In a prune-based approach ($n - 1$) points of generic multi-point P pruned and 1 point remains that is the real position of node i .

Consider $i \in \{1, 2, \dots, n\}$ as a sample sensor node of the network, $P(i)$ stands for the position of node i and $K(i) : \{1, 2, \dots, n\} \rightarrow R^2$ shows the function that estimates position of node i . The estimation error of function K for node i is calculated using Equation 2 where $error(i)$ is the error of function K for node i .

$$error(i) = \|P(i) - K(i)\| \quad (2)$$

Total average position error of function K for network with graph G can be calculated using Equation 3.

$$Mean_{error} = \frac{1}{n} \sum_{i=1}^n (\|P(i) - K(i)\|) \quad (3)$$

$E(i)$ represents the remaining energy of node i while $E_{initial}$ is the initial energy of a node. $E_{used}(i)$ shows consumed energy for localization of node i .

$$E(i) = E_{initial} - E_{used}(i) \quad (4)$$

According to Equation 4, average remaining energy of the network can be obtained from Equation 5.

$$Mean_{remaining_energy} = \frac{1}{n} \sum_{i=1}^n E(i) \quad (5)$$

Response time for localizing network nodes is represented by $T(n)$.

$$response_time = T(n) \quad (6)$$

The localization algorithm should calculate the positions of nodes with the goal of minimizing average position estimation error. Additionally, the response time of the process and the consumed energy to fulfil the task should be bounded to a certain limit.

4 Non-directional Bilateralation Position Estimation

In NBPE, nodes can be in either states of *unknown* or *anchor*. Unknown state refers to the nodes unaware of their location. Anchor state refers to nodes that know their locations. A node may switch its state from *unknown* to *anchor* as soon as it finds its location.

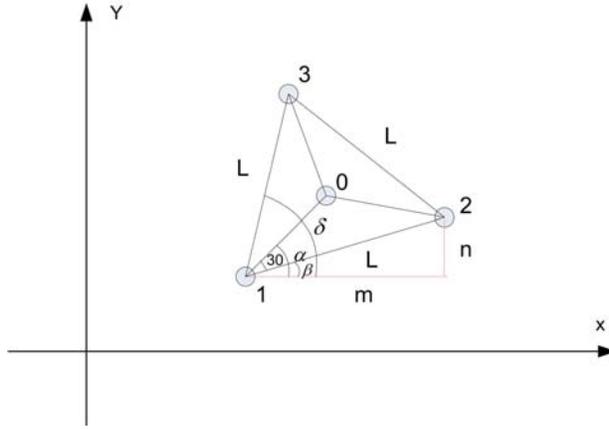
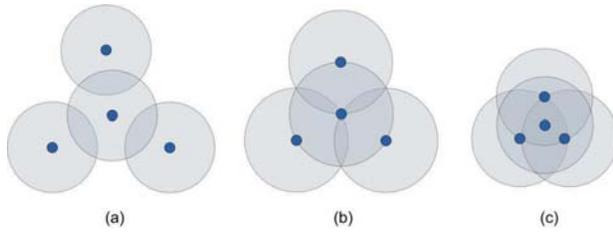
NBPE uses a prune-based approach launching from a point and proceeding recursively. The approach possess the localizability of the whole nodes in bilateralation networks. The approach utilizes a criterion named *common anchor* node, to prune the invalid positions. Given v_i as an *unknown* node with two neighbouring anchor nodes v_j and v_k , the common anchor node which is in the vicinity of both v_j and v_k . The way to implement pruning among two possible positions is nearness to the common anchor node. Hence, there are no specified localization directions, since common anchor nodes are continuously changing during the process.

The primary anchor nodes in NBPE follows a equilateral triangle (Figure 6). The reason to use this structure is to equip only two of them with GPS receivers, and calculating two other nodes position accurately.

Based on Figure 6 and using the following equations, the locations of node 2 and 3 can be determined precisely.

$$\alpha = \arctan\left(\frac{y_1 - y_0}{x_1 - x_0}\right) \quad (7)$$

$$x_2 = x_1 + m, \quad m = L \cdot \cos \beta, \quad \beta = \alpha - 30 \quad (8)$$


Figure 6: Primary anchors structure.

Figure 7: Coverage areas. (a) Distance is more than t_r . (b) Distance is equal t_r . (c) Distance is less than t_r .

$$y_2 = y_1 + n, \quad n = L \cdot \sin \beta \quad (9)$$

It can be derived from the above equations that:

$$x_2 = x_1 + L \cdot \cos(\alpha - 30) \quad (10)$$

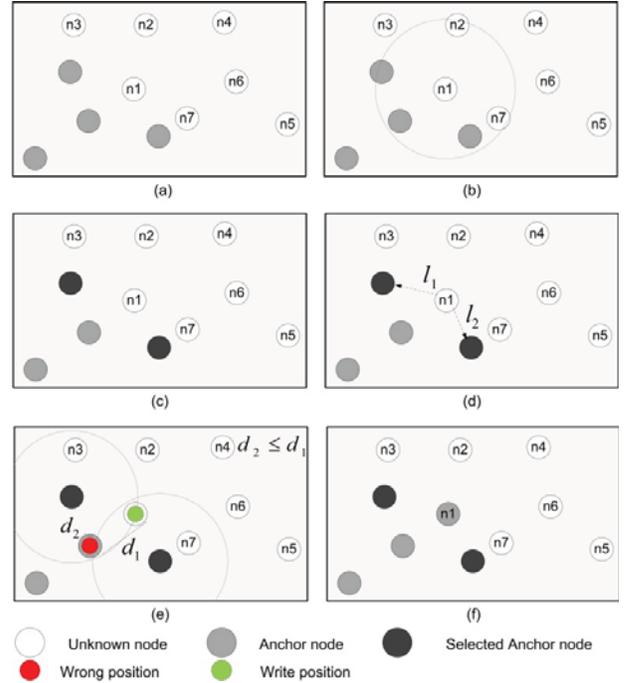
$$y_2 = y_1 + L \cdot \sin(\alpha - 30) \quad (11)$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 + L \cdot \cos(\alpha - 30) \\ y_1 + L \cdot \sin(\alpha - 30) \end{bmatrix} \quad (12)$$

$$\begin{bmatrix} x_3 \\ y_3 \end{bmatrix} = \begin{bmatrix} x_1 + L \cdot \cos(\alpha + 30) \\ y_1 + L \cdot \sin(\alpha + 30) \end{bmatrix} \quad (13)$$

The distance (L) from nodes 1, 2, and 3 to node 0 in this structure is set to be equal to t_r (nodes radio range). This will enable NBPE to start localization process with primary anchor nodes, that means maximizing coverage by at least two anchor nodes (to better launch localization) as well as coverage by just one anchor node (to limit primary error propagation). If the distance (L) is more than t_r , coverage by at least two anchor nodes will decrease (Figure 7.a) and if the distance (L) is less than t_r , coverage by just one anchor node decreases, while coverage by at least two anchor nodes will not change (Figure 7.c). Therefore, in comparison to the states explained, considering the distance equal to t_r has the advantage compared to others (Figure 7.b).

NBPE utilizes a precedence mechanism on anchor nodes to district irregular error propagation through the network. Each *unknown* node selects those


Figure 8: Main steps in NBPE. (a) Node 1 intends to localize itself. (b) Identifies all neighbouring anchor nodes. (c) Selects two qualified anchor nodes. (d) Measures the distances l_1 , l_2 to them. (e) Calculates two possible location and selects the one farther from common anchor node. (f) Turns into an anchor node and broadcasts its location.

two neighbouring anchor nodes having the highest precedence to localize itself. The precedence mechanism is based on the precedence number. The precedence number for primary anchor nodes is considered 1 and having turned into an anchor node, each *unknown* node obtains its precedence number equalling to sum of two selected anchor nodes precedence number. At a moment if a number of unknown nodes find enough anchor node for self localization, NBPE algorithm do not use any precedence mechanism for their localization priority. This may cause localization error increase. Precedence mechanism implementation among unknown nodes is obliged to be practised either through packet exchange between them or through time scheduling. The former causes energy consumption increase and the latter makes total response time rising.

As illustrated in Figure 8, NBPE algorithm is composed of 6 steps.

Step 1: An anchor node broadcasts its position. The message consists of the anchor node ID, precedence number, sending signal power, the anchor position information, and neighbourhood information including the list of nearby anchors and their locations.

Step 2: Unknown nodes identify all neighbouring anchor nodes provided that the received signal power be higher than a predetermined strength threshold.

Step 3: A pair of anchor nodes of lower precedence number are selected along with a common anchor node. If there would be more than one qualified pair of anchor nodes, the pair having lower precedence number is selected. To find out the common anchor node for the pair, an *unknown* node intersects the list of nearby anchor nodes.

Step 4: Using the RSSI technique the distance from an *unknown* node to the selected pair of nodes is estimated by Friss (Rappaport et al. 1996) model.

Step 5: Two possible positions for an *unknown* node is calculated by equation of two circles intersection, each circle having one selected anchor node as its centre and farther of them from common anchor node is selected as valid position.

Step 6: The node turns into an anchor node and broadcasts its position.

Regarding that localization error increases as the process continues, next anchor nodes from which signals are received, absolutely have more imprecise position information and also higher precedence number. Therefore, each anchor node broadcasting its position earlier is of higher priority (lower precedence number) and more precise position information.

Algorithm 1:

```

1: procedure NBPE-ESTIMATE
2:   sort(neighbors);
3:    $i = 0$ ;
4:    $j = 1$ ;
5:   while ( $i < \text{NumberOfNeighbors}$ ) do
6:      $\text{commonAnchor} = \text{findComAnchor}(i, j)$ 
7:     if ( $\text{commonAnchor} \neq \text{NULL}$ ) then
8:        $\text{calcPosition}(\text{nb}[i], \text{nb}[j], \text{commonAnchor})$ ;
9:        $\text{isReference} = \text{TRUE}$ ;
10:       $\text{broadcast}(\text{location})$ ;
11:      exit;
12:     else
13:        $j++$ ;
14:       if ( $j \geq \text{NumberOfNeighbors}$ ) then
15:          $i++$ ;
16:          $j = i$ ;
17:       end if
18:     end if
19:   end while
20: end procedure

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Algorithm 1 presents the steps of NBPE. In line 2, an *unknown* node sorts all the nearby anchors based on precedence number. In line 3 to 20 pair of neighbouring anchors is selected. Based on the selected pair the positions of the *unknown* node are estimated. In line 6, it is considered that whether each pair anchor nodes have common neighbouring anchor nodes or not. If the condition takes place in line 8, the *unknown* node calculates two possible positions by using intersection of the two circles centred by neighbours named $\text{nb}[i]$ and $\text{nb}[j]$ and selects the farther one from

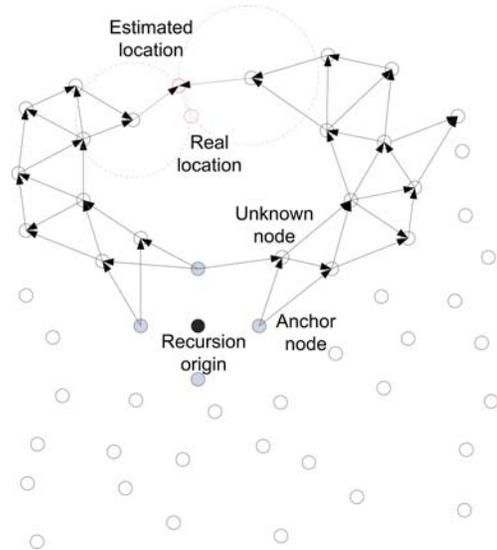


Figure 9: DPE wrong position estimation caused by communication hole.

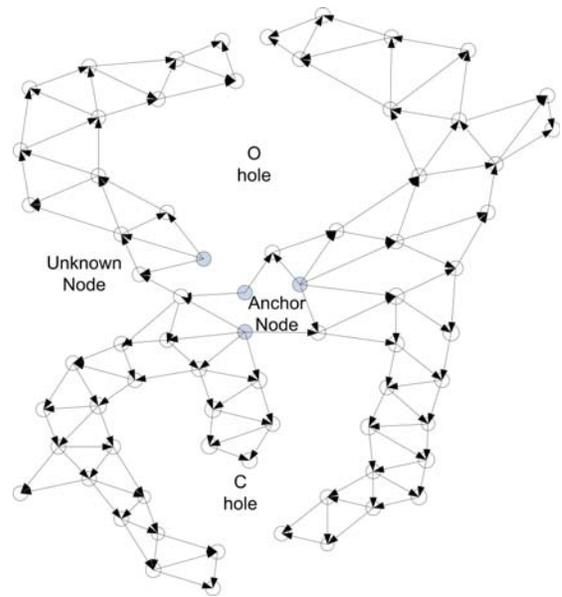


Figure 10: NBPE confronting against *O* and *C* shaped holes.

the *CommonAnchor* node. In line 9, it turns into anchor and in line 10 it broadcasts its position.

Hole existence in some localization approaches decreases the number of localized nodes (Rahman & Kleeman 2009, Albowicz et al. 2001, Aspnes et al. 2006) and in some others increases localization error (de Oliveira et al. 2009, Nagpal et al. 2003, Niculescu & Nath 2003b, Zheng et al. 2012). Figure 9 illustrates how it deteriorates localization accuracy in DPE (de Oliveira et al. 2009). Since pruning criterion in this approach is nearness to the recursion origin, it does not act correctly for the node over the hole and this causes a

wrong estimation. In NBPE changing pruning criterion from recursion origin to common anchor node prevents incorrect position estimation. On the other hand since NBPE requires low necessities (bilateration networks) for nodes localization and is recursively implemented, hole existence has little influence on the number of located nodes and controls the increase on position estimation error. Figure 10 shows how NBPE confronts two O and C like shaped (Yang & Liu 2012, Yang et al. 2009) holes.

5 Evaluation

The performance analysis of the devised approach is performed from various aspects. First, the computational complexity and the overheads of the approach are discussed. Then, NBPE performance is analysed using NS2 simulation and the results are compared with RPE (Albrowicz et al. 2001) and DPE (de Oliveira et al. 2009) algorithms. Finally, the effect of RSS inaccuracy and node density on NBPE is studied.

5.1 Complexity and Overhead

NBPE launches localization by a primary anchor nodes structure and each node broadcasts its position after localizing itself in order to be used by neighbouring nodes with *unknown* status. Therefore, each node receives and sends one packet. Consequently, it can be claimed that the communication complexity of NBPE is $O(n)$, where n indicates the number of network nodes.

Recall from previous section, the location of nodes are calculated by means of two circle intersection centred at anchor nodes. Two circle intersection equation consists of simple calculation taking 20 flops per node (de Oliveira et al. 2009). However, the complexity does not end to this, there are imposed additional costs including search and selection of two proper anchor from the neighbouring nodes. This act is implemented by sorting neighbouring anchor by their precedence numbers. Sorting cost equals to $O(k \cdot \log k)$, where k determines neighbouring anchors number.

5.2 Modelling upper bound of position estimation error

Supposing that anchor nodes are placed at the centre of sensor field, the farthest point to the primary anchors will be on the corners of rectangle (sensor field). The distance between the centre of the rectangle and its corners will be $\frac{\sqrt{a^2+b^2}}{2}$ where a and b demonstrate length and width of the rectangle. As we benefit precedence mechanism in NBPE maximum localization error on corner nodes will equal to the least rounds from the centre multiplied by the error(i).

Suppose that $max_{error}(i)$ demonstrates maximum position estimation error of the first unknown node and min_{rounds} shows the minimum localization rounds from

the centre of sensor field to its corner calculated from Eq. 14, then the upper bound of the localization error can be determined by Eq.15.

$$min_{rounds} = \lceil \frac{\sqrt{a^2 + b^2}}{2t_r} \rceil \quad (14)$$

$$error_{upperbound} = min_{rounds} \times max_{error}(i) \quad (15)$$

5.3 Performance analysis

Several simulation scenarios are designed to analysis the performance of the proposed approach using network simulator NS-2. NS-2 is one of the most widely used tools to analyse WSN protocols and services. It has the basic properties and capabilities to support simulations of different localization techniques. A positioning system framework implemented using NS-2 developed by M.Abu-mahfuze (Abu-Mahfouz et al. 2012) is used for simulating of WSN and evaluating the localization approaches. It modelled the free-space path loss and fading, multi-hop propagation technique and RSS technique for range measurement. The evaluation parameters are localization accuracy, response time, nodes remaining energy level, effect of RSS inaccuracy on localization error, and node density on the number of localized nodes has been surveyed as well. The results are compared with RPE (Albrowicz et al. 2001) and DPE (de Oliveira et al. 2009) algorithms.

In the simulation configuration, the communication range of nodes are set to 45 meters. To simulate noise, each measured distance is disturbed by a normal distribution with the actual distance as the mean and %5 of this distance as the standard deviation. The nodes are distributed randomly in a $200 \times 200 m^2$ field. Each experiment is repeated 100 times to reduce the effect of randomness. The primary anchor nodes structure for DPE (de Oliveira et al. 2009) and NBPE are placed in the centre of the network area. In DPE and RPE approaches 4 anchor nodes and in NBPE, 2 of them are used.

Connectivity degree of nodes is obtained by following equation:

$$Connectivity\ degree = \frac{n \times S_i}{S_{net}} \quad (16)$$

In Equation 16, n shows nodes number, S_i determines coverage area by each sensor node and S_{net} reminds network coverage area. Connectivity degree illustrates each node's average connection number with neighbouring ones. Every approach is studied in three different connectivity degrees as 6.28 ($n = 40$), 15.89 ($n = 100$) and 23.84 ($n = 150$) periodically representing sparse, normal and dense networks.

5.3.1 Localization error

Localization error is one of the performance parameters analysed for NBPE. The parameter is measured based on Equation 3, that determines in-time total average

position error of the network i.e. while each new node being localized, total average position error for every localized node until that time is calculated.

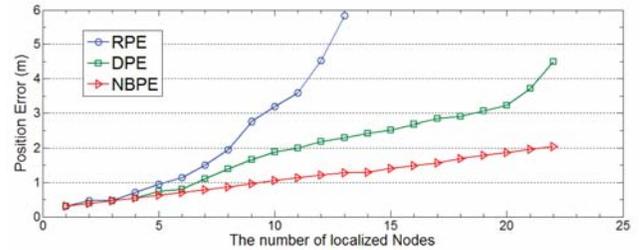
Three different simulation scenarios are configured for various connectivity degrees and the results are compared with RPE (Albawicz et al. 2001) and DPE (de Oliveira et al. 2009) algorithms. Figure 11 illustrates how error propagation occurs in NBPE as well as the two other approaches. The figure shows that error rates in NBPE have a lower slope which leads to the decrease in the total propagation error. Among the three simulation scenarios the amount of improvement is more significant in the networks with higher connectivity degree (Figure 11c). The reason is that the less the connectivity degree is, the less the choices unknown nodes will have in their neighbouring anchor nodes, resulting in more similarity in NBPE and DPE approaches function as shown in Figure 11a. However, the increase in connectivity degree results in more anchor selection options for unknown nodes ending in the better control of irregular error propagation shown in Figure 11c. In some points DPE experiences a sudden mutation hole existence and wrong position estimation causes the sudden mutations occurring more frequently in lower connectivity degrees where NBPE prevents major error occurrence by waiting and not positioning the node.

5.3.2 Energy efficiency

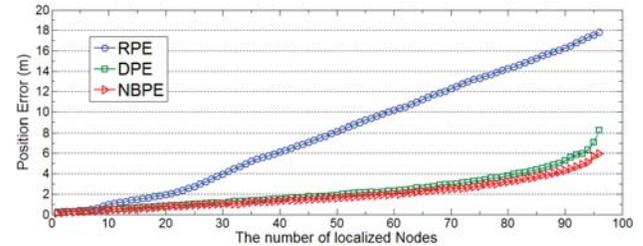
Energy consumption rate is one of the most important evaluation parameters in most of WSN protocols. Different parts of a sensor node involves energy consumption. Equation 5 is used to calculate in-time average remaining energy. Initial energy of each node is set to 2 joules. In Figure 12, the X axis shows i^{th} localized node and the Y axis determines in-time average remaining energy level in joule. Energy consumption difference between DPE and NBPE is caused by additional process to find proper pair of anchor node in NBPE. In lower connectivity degrees (Figure 12a) the difference is close to zero because unknown nodes have lower choices, hence have lower processing overhead. In the dense network scenario, more neighbouring anchor nodes results in more processing overhead to select proper pair of anchors. Subsequently, energy consumption increases (Figure 12c), however the difference can be ignored (approximately 0.002j).

5.3.3 Response Time

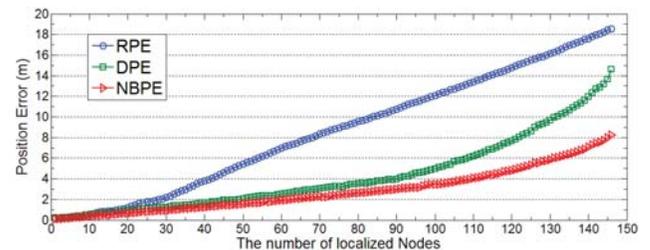
Response time another evaluation parameter for localization algorithms. To calculate the parameter Equation 6 is applied. In Figure 13, the X axis shows i^{th} localized node and the Y axis illustrates in-time response time. As the figure shows DPE has a lower response time in comparison with NBPE, here every unknown node does not check any condition to select pair of neighbouring anchor node, so it performs faster. NBPE response time depends directly on the number



(a) Connectivity degree: 6.28.



(b) Connectivity degree: 15.89.



(c) Connectivity degree: 23.84.

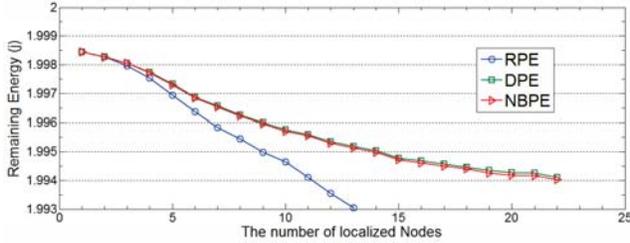
Figure 11: Positioning error average.

of neighbouring anchor nodes. Therefore, in higher connectivity degrees the time to select proper pair of anchor node and as a result the response time increases (Figure 13c). But in lower connectivity degrees, the response time decreases and nears to DPE (Figure 13a). However, the difference between two approaches response time even in higher connectivity degrees is low (2.4 Sec) and ignorable according to NBPE improvement in localization error comparing DPE (6.5 Min).

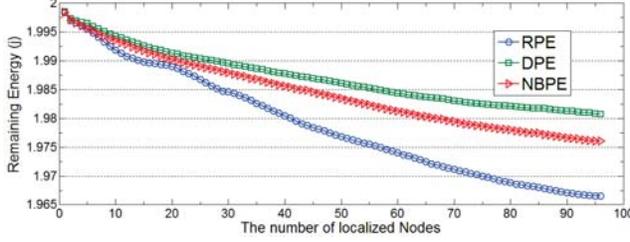
5.3.4 The Impact of RSS inaccuracy on localization error

Measurement distances by means of RSS is not absolutely precise. In noisier environments distance measurement errors become more inaccurate leading to localization error increase. To simulate noise, each measured distance is distributed by a normal random variant [47] with the following settings: a mean of actual distance between two nodes and a standard deviation of %1 to %25 of the actual distance.

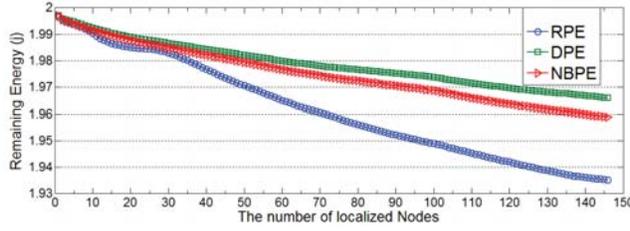
Simulations for the network is done in connectivity degree 20 (calculated with Equation 16). In Figure 14, the X axis shows amount of RSS noise and the Y axis determines average localization error in network. In both DPE and NBPE localization error increases linearly by



(a) Connectivity degree: 6.28.



(b) Connectivity degree: 15.89.



(c) Connectivity degree: 23.84.

Figure 12: Residual energy level.

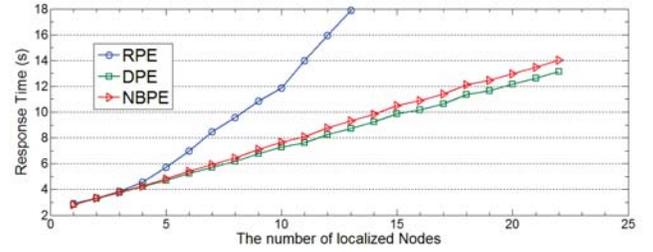
RSS error increase, yet in every point NBPE has lower localization error in comparison to other approaches.

5.3.5 The Impact of nodes density on number of localized nodes

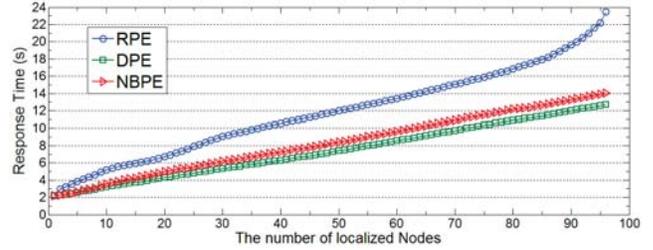
A lot of localization algorithms performs well just in specific densities (usually higher densities), but the proper algorithm is the one that performs so well in a wider range of densities from sparse to dense. To represent various densities, usually two equations are used. The first, formerly mentioned in formula 16. that shows average number of connections for each node with neighbouring nodes. The second one (Equation 17) illustrates number of nodes per square meter whose unit is $\frac{\text{number of nodes}}{\text{m}^2}$.

$$\text{node density} = \frac{n}{S_{net}} \quad (17)$$

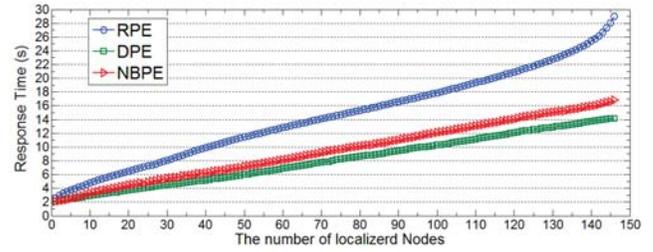
The problem here is that radio range of sensor nodes is not considered, so we use the connectivity degree calculated through Equation 16. Figure 15 shows how connectivity degree increase from 4.768 to 20.665 in the X axis affects on localized nodes percent in the Y axis. According to results in, hand the nodes number that DPE and NBPE localize are almost the same and they localize larger number of nodes in comparison to



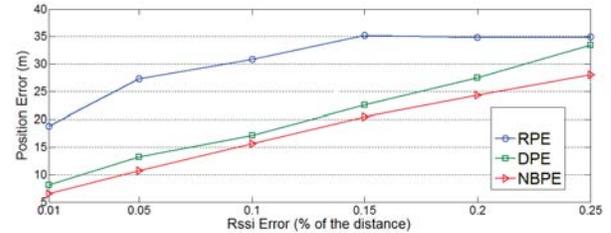
(a) Connectivity degree: 6.28.



(b) Connectivity degree: 15.89.



(c) Connectivity degree: 23.84.

Figure 13: Response time.

Figure 14: Influence of RSSI error on positioning error.

RPE in every connectivity degree. It is because in the approaches each unknown node needs fewer requirements (two anchor nodes) to localize itself.

It is inevitable in most of the recursive approaches to prevent localization error increase during the process, but the success will be of the approach having ability to control Irregular error propagation. In NBPE, it is because of utilizing a kind of precedence over anchor nodes that controls the propagation of error throughout the network properly.

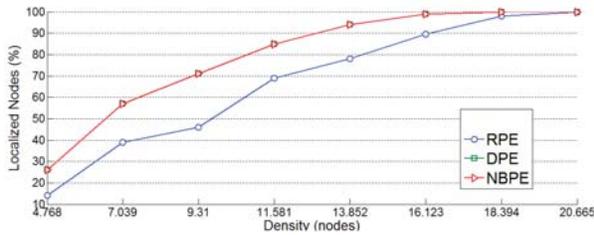


Figure 15: Influence of network density on number of localized nodes.

6 Conclusion

This paper has presented a prune based recursive approach, NBPE, for localization in WSNs. Every unknown node can localize itself using only two anchor nodes that increase the number of localizable nodes comparing with Trilateration technique. On the other hand to reduce the costs, only two of four nodes in central structure are equipped with GPS and the locations of other two are calculated precisely. Hole existence in DPE causes wrong location estimation. To solve this, another criterion, named common anchor node, is utilized to prune wrong position in NBPE. Simulations results done for sparse, normal, and dense networks show how our proposed approach, NBPE, has lower localization error in comparison with two other approaches, DPE and RPE, in despite of ignorable increase in energy consumption and response time. Also experiments show in different noisy environments, NBPE maintains its advantage.

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