

سوال ۱:

$$\frac{h\nu}{c} = \gamma m_0 u \rightarrow \text{پای تکیه}$$

$$h\nu + m_0 c^2 = \gamma m_0 c^2 \rightarrow \text{پای انرژی}$$

$$\rightarrow h\nu = \frac{h\nu}{cu} c^2 - m_0 c^2 \Rightarrow \frac{h\nu c}{u} = h\nu + m_0 c^2 \Rightarrow \boxed{u = \frac{h\nu c}{h\nu + m_0 c^2}}$$

پای تکیه  
پای انرژی

$$m = \gamma m_0$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{(h\nu)^2 c^2}{(h\nu + m_0 c^2)^2 c^2}}} = \frac{1}{\sqrt{\frac{(h\nu + m_0 c^2)^2 - h^2 \nu^2}{(h\nu + m_0 c^2)^2}}}$$

$$= \frac{h\nu + m_0 c^2}{\sqrt{m_0^2 c^4 + 2 m_0 c^2 h\nu}} = \frac{c^2 \left( \frac{h\nu}{c^2} + m_0 \right)}{c^2 \sqrt{m_0^2 + \frac{2 m_0 h\nu}{c^2}}}$$

$$\Rightarrow m = \frac{\frac{h\nu m_0}{c^2} + m_0^2}{\sqrt{\frac{2 m_0 h\nu}{c^2} + m_0^2}} \approx \left( m_0^2 + \frac{2 m_0 h\nu}{c^2} \right)^{1/2}$$

سوال ۲:

$$\frac{\lambda}{\lambda_0} = \left( \frac{1 - \beta}{1 + \beta} \right)^{1/2}$$

$$\lambda_{\text{green}} = \lambda = 5 \times 10^{-5} \text{ cm} , \lambda_{\text{red}} = \lambda_0 = 7 \times 10^{-5} \text{ cm}$$

$$\Rightarrow \frac{5}{7} = \left( \frac{1 - \beta}{1 + \beta} \right)^{1/2} \Rightarrow \frac{25}{49} = \frac{1 - \beta}{1 + \beta} \Rightarrow (1 + \beta) \frac{25}{49} = 1 - \beta$$

$$\Rightarrow \beta = \frac{24}{74} = 0.32 \Rightarrow \boxed{v = 0.32 c}$$