بسم الله الرحمن الرحیم

Advanced Quantum Mechanics, 3rd assignment, Due date: moments before the final exam starts!

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1. Try to derive the Green function of the operator

$$\mathbf{L} := -\frac{\hbar^2}{2m}\nabla^2 + V(\vec{\mathbf{x}}) - i\hbar\partial_t.$$

2. Show by detailed calculations that

$$[\,\Pi_i, \Pi_j\,] = i\varepsilon_{ijk}\frac{\hbar e}{c}B_k,$$

where $\mathbf{\Pi}$ and $\mathbf{B}$ are the kinematical momentum and the magnetic field, respectively.

3. Find the Lorentz force, using the Heisenberg picture,

4. Derive

$$-\frac{1}{Z}\partial_\beta Z$$

and interpret the result, where $\beta := it/\hbar$ and $Z := \int dx' K(x', t; x', 0)$.

5. Determine the elements of the operator U defined by

$$U := \langle A|\Sigma\rangle/\langle\Sigma|A\rangle,$$

where $A = (a_0, a_1, a_2, a_3)$ is an arbitrary real vector, $\Sigma := (I, i\vec{\sigma})$ is a 2×2-matrix-valued vector with I and the components of $\vec{\sigma}$ representing the identity and Pauli's matrices respectively, and $\langle .|. \rangle$ is the traditional sesquilinear inner product but now in four-dimensional space.

Good Luck
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