بسم الله الرحمن الرحیم

Advanced Quantum Mechanics, 2nd assignment, Due date: Azar 15 1401

Consider the state

$$|\lambda\rangle = C\Sigma_{n=0}^{n=\infty} \frac{\lambda^n}{\sqrt{n!}}|n\rangle$$

where $|n\rangle$ denotes the n-th state of a 1D simple harmonic oscillator and, λ and C are complex numbers.

1. Compute
$$a|\lambda\rangle$$
and show that $|\lambda\rangle$ is an eigenvector of a, where a is the annihilation operator. Find the corresponding eigenvalue.

2. Find the normalization constant C.

3. Evaluate $\langle x\rangle_\lambda$ and $\langle p\rangle_\lambda$.

4. Try to examine the uncertainty relation between x and p for the state $|\lambda\rangle$.

5. Show that
$$e^{\lambda(a-\lambda/2)^\dagger}|0\rangle$$
is $|\lambda\rangle$.

6. Prove that by applying the finite translation operator to the ground state, one can derive $|\lambda\rangle$.

Good Luck
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