بسم الله الرحمن الرحیم

Advanced Quantum Mechanics, 1st assignment, Due date: Aban 22 1401

1. In an inner product space, prove that

$$\|\mathbf{u}+\mathbf{v}\| \leq \|\mathbf{u}\| + \|\mathbf{v}\|.$$

2. In $\mathbb{R}^2$, construct the projection operator that maps any vector to the line $y = x$.

3. Show that for any operator A,

$$[A, F(A)] = 0,$$

where $F(A)$ is an arbitrary function of A.

4. Show that for any operators A and B with $[A, B] = cI$, where c and I are a complex number and the identity element respectively,

$$e^{A} B e^{-A} = B + [A, B]$$

5. If $[A, B] = [A, C] = 0$ for three operators A, B and C, does it yield $[B, C] = 0$? Why?

Good Luck
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