بسم الله الرحمن الرحيم

Electrodynamics, 3rd Homework assignment, Fall 1402, Due date: Dey 23

1. A magnetic dipole $m = -m_0\hat{\mathbf{z}}$ is situated at the origin, in an otherwise uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$. Show that there exists a spherical surface, centered at the origin through which no magnetic field lines pass. Find the radius of this sphere, and sketch the field lines, inside and out.

2. When the velocity of charges in a conductor is not ignorable, the Ohm's law is generalized to $\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B})$, where σ is the conductivity of the medium. Show that for a perfect conductor with finite $\mathbf{J}$, $\partial\mathbf{B}/\partial t = \nabla \times (\mathbf{v} \times \mathbf{B})$.

3. A magnetostatic field is due entirely to a localized distribution of permanent magnetization. Show that $\int d^3x \mathbf{B} \cdot \mathbf{H} = 0$, provided the integral is taken over all space.

4. Superconductors exclude magnetic fields from their interiors. While conductors exclude electric fields by having a surface charge, superconductors exclude magnetic fields by having a surface current.
   (a) Show that boundary conditions on the magnetic field imply that, just outside the superconductor,
   $$\mathbf{B} \cdot \hat{\mathbf{n}} = 0 \tag{1}$$
   where $\hat{\mathbf{n}}$ is the normal to the surface of the superconductor.

   Consider putting a spherical superconductor of radius R in a constant magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$. Since outside the superconductor $\nabla \times \mathbf{B} = 0$, one can write $\mathbf{B} = -\nabla\Phi_m$, where Φ_m satisfies Laplace equation.
   (b) Write down the boundary conditions on Φ_m at infinity and on the surface of the sphere.
   (c) Find Φ_m for $r > R$ using Legendre polynomials.
   (d) Find $\mathbf{B}$ everywhere.
   (e) From the boundary condition on $\mathbf{B}$, find the surface current (magnitude and direction, as a function of θ at $r = R$ on the superconductor. Draw the approximate field lines everywhere in the space.

5. A conducting sphere at potential V_0 is half embedded in a linear dielectric material of susceptibility ξ_e, which occupies the region $z < 0$ as shown in the sketch.
   (a) Find the potential $V(r)$, in terms of V_0, R, and r, everywhere.
   (b) Find the field, the polarization and the bound charge everywhere outside and on the sphere.
   (c) Find the free charge distribution on the sphere.
   (d) Find the total free charge on the sphere.

Good Luck
H. Shojaie

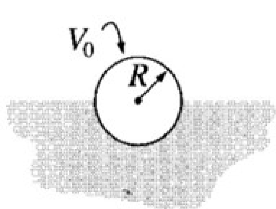

Figure 1: Problem 5 (Griffiths J.D., 1999, Introduction to Electrodynamics)