بسم الله الرحمن الرحیم

Electrodynamics, 2nd Homework assignment, Fall 1402, Due date: Azar 28

1. A sphere of radius R has a certain charge density distribution $\sigma(\theta)$. There are no other charges. The potential on the sphere is
$$V = V_0 + V_1 \cos\theta + V_2 \sin^2\theta,$$
where V_0, V_1, and V_2 are constants.
   (a) Find $V(r,\theta)$ everywhere for $r < R$ and $r > R$.
   (b) Find $\sigma(\theta)$.

2. A hollow right circular cylinder of radius a has its axis coincident with the z-axis and its ends at $z = 0$ and $z = L$. The potential on the end faces is zero, while the potential on the cylindrical surface is given by $V_0 \sin 2\varphi$.
   (a) Find the potential everywhere in the cylinder.
   (b) Determine the potential on the z-axis inside the cylinder.

3. Two infinitely long coaxial conducting cylindrical shells with radii a and b ($b > a$), are at potentials $V_a(\varphi, z)$ and $V_b(\varphi, z)$, respectively.
   (a) Find an expression for the potential at any point between the two cylinders.
   (b) How many independent coefficients should be determined? Try to write down an expression for at least one of them.

4. Two long, cylindrical conductors of radii a_1 and a_2 are parallel and separated by a distance d, which is large compared with either radius. Show that the capacitance per unit length is given approximately by $C \simeq \pi\epsilon_0 \left(\ln \frac{d}{a}\right)^{-1}$, where a is the geometrical mean of the two radii.

5. The potential $V(\theta) = V_0 \cos\theta$ is specified on the surface of a sphere of radius a centered on the origin. By applying an appropriate Green's function, find the potential on the z-axis (both outside and inside the sphere). Hint: the following integral may be helpful:
$$\int d\theta \frac{\sin\theta\cos\theta}{(A + B\cos\theta)^{3/2}} = -\frac{4A + 2B\cos\theta}{B^2 (A + B\cos\theta)^{1/2}}.$$

6. Prove that the potential at the center of a regular polyhedron whose sides are kept at different potentials V_i, is the average of V_i's.

Good Luck
H. Shojaie