A WAVELET PACKET BASED MODEL  
FOR AN ULTRA-WIDE-BAND INDOOR PROPAGATION CHANNEL  

Seyed Mohammad Sajad SADOUGH, Emmanuel JAFFROT  

Ecole Nationale Supérieure de Techniques Avancées, 32 Bd Victor, 75739 Paris cedex15,  
France  
Email: Sajad.Sadough@ensta.fr, Emmanuel.Jaffrot@ensta.fr  

ABSTRACT  
A sparse model for ultra-wide-band (UWB) indoor channel impulse response (CIR) based on wavelet packets is presented. The CIR is decomposed on wavelet packet basis functions resulting in scaling and wavelet coefficients. Hard thresholding is used to compress these coefficients as much as possible. Different threshold levels have been tested to find the best trade off between the mean square error (MSE) due to the reconstructed CIR after compression and the compression rate (CR). Finally, the robustness of the model to small-scale variations of the CIR has been checked by the estimation of the mean and variance of model performances and satisfactory results have been obtained.  
Keywords: UWB, Channel modelization, wavelet packet transform, rank reduction.  

1 INTRODUCTION  
A new technology in wireless communication known as Ultra-wide-Band (UWB) technology is currently receiving a great deal of attention. In February 2002, the Federal Communications Commission (FCC) allocated 7500 MHz of spectrum for unlicensed applications in the 3.1-10.6 GHz frequency band. This spectral allocation has started an extremely productive research activity for both industry and academia. A UWB radio signal is defined as any signal whose bandwidth is at least 25% of its center frequency or greater than 500 MHz [1]. Because of multipath fading as a source of intersymbol interference in wireless systems, the propagation channel must be well identified to enable the receiver deal with channel estimation and equalization if necessary. Channel modeling has been studied extensively in the past but no major work has been done to develop a sparse channel model with reduced parameters.  
In [3], different probability density functions are proposed for the arrival time sequence, path amplitudes and path phases of the indoor channel impulse response (CIR). For example amplitude fading in a multipath channel may follow different distributions such as Rayleigh distribution depending on the environmental conditions of the receiver antenna.  
Although statistical channel models can precisely characterize the propagation channel, their parameters depend on channel statistics, making them not appropriate for implementation issues. On the other hand, in real-time digital transmission systems, a sparse channel model with few parameters to estimate or transmit is of great interest. UWB channel modeling suffers from a lack of literature in terms of reduced parameter methods [3]. This is the motivation for finding a decomposition basis for CIR which would be independent of the propagation environment.  
In this paper we propose a new model using wavelet packets as a decomposition basis for UWB indoor CIR. Two types of CIR profiles are considered: in the first scenario Tx and Rx antennas are in direct line of sight (LOS) while in the other they are not in LOS (NLOS) [4].
This paper is organized as follows: section 2 introduces the proposed modeling procedure. In section 3 we analyze the robustness this new model to CIR small-scale variations. Simulation results and a comparison of the model performances between our proposed model and another UWB channel model is made in section 4 and finally section 5 concludes the paper.

2 MODELING PROCEDURE

Let $h$ be the CIR vector of a UWB propagation channel and $w_{m,j}(m,j)\in N^2$, the wavelet packet basis functions. Our model is based on wavelet packet transform (WPT) of $h$ defined as [2]:

$$C_{j,m}[k]=\left<h(n),\frac{1}{2^j}w_m(n)\right>, n\in N, K\in Z$$ (1)

where $C_{j,m}[k]$ are the wavelet packet coefficients.

The next step consists in compressing these coefficients by hard thresholding as follows [2]:

$$C_{j,m}^c[k]= \begin{cases} C_{j,m}[k], & \text{if } |C_{j,m}[k]|\geq \varepsilon \\ 0, & \text{elsewhere} \end{cases}$$ (2)

where $C_{j,m}^c[k], k\in Z$ are the compressed wavelet packet coefficients and $\varepsilon$ is the applied threshold level. Because of orthogonal nature of wavelet decomposition, the CIR may be reconstructed by the inverse wavelet packet transform (IWPT). In order to have less wavelet packet coefficients for the model, the reconstructed CIR vector ($h_r$) is derived from compressed coefficients by the following relation [2]:

$$h_r = \sum_j \sum_m C_{j,m}^c w_{m,j}$$ (3)

Model performances are evaluated via mean square error (MSE) as a result of thresholding and compression rate (CR) as defined in the following:

$$\text{MSE} = \frac{1}{N} \sum_{n=1}^{N} |h(n) - h_r(n)|^2 ,$$ (4)

$$\text{CR} = \frac{N_j}{N} \times 100 .$$ (5)

where $N$ is the length of CIR vector and $N_j$ is the number of wavelet packet coefficients replaced by 0 after thresholding. The entropy coefficient, defined as 100-CR, may also be used for model performance evaluation. The followed objective is to increase as much as possible the CR but one must know that attempting to increase too much the CR raises the MSE and leads to an imprecise model. Therefore we need to find an optimal threshold level defining the best trade off between CR and MSE. Fig. 1 schematically shows our modeling scheme.

3 ROBUSTNESS OF THE MODEL TO CIR SMALL-SCALE VARIATIONS

Our database consists in 440 CIR profiles corresponding to different positions of Rx antenna (distants of a few centimeters) in the same “local area”. Small-scale variations refer to the minute variations between the impulse responses collected in the same site. This is due to the fact that the behaviour of the channel does not change appreciably over short distances. A model is said to be robust if its parameters are not very sensitive to small-scale variations of CIR. It is interesting to study the variations of MSE and CR with respect to different CIR profiles. We estimate the mean and variance of MSE and CR relative to different CIR profiles by an empirical method. In order to analyze the effect of different threshold (ε) levels on model performance, we simulate the model using various thresholds and estimate the mean and variance of the MSE and coefficient’s entropy as a function of ε. It is obvious that the MSE mean is an increasing function while the mean of the entropy coefficient is a decreasing function of ε and their respective curves intersect each other. Indeed the best trade off between MSE and CR is obtained by taking the threshold level corresponding to the intersection point of the two functions. We tested this method on both LOS and NLOS CIR and recorded respective optimal threshold values.

Fig. 2 illustrates the variations of mean and variance of MSE and the entropy coefficients with respect to different thresholds in the case of symmetric wavelet packet family. Table 1 compares the obtained results in the case of a LOS CIR for a model using symmetric, coiflet and biorthogonal wavelet families.

As this table shows, symmetric wavelet family leads to smallest performance fluctuations and provides smallest variances increasing the robustness of the model with respect to CIR variations. Similar results have been observed for NLOS channel configuration.
Fig. 2. Mean and variance of MSE and Entropy for symmetric wavelets for a LOS channel

<table>
<thead>
<tr>
<th>Wavelet family</th>
<th>Mean of MSE (%)</th>
<th>Mean of CR (%)</th>
<th>Variance of MSE (%)</th>
<th>Variance of CR (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetric</td>
<td>5.74</td>
<td>94.26</td>
<td>0.19</td>
<td>0.54</td>
</tr>
<tr>
<td>Biorthogonal</td>
<td>5.98</td>
<td>94.02</td>
<td>0.24</td>
<td>0.56</td>
</tr>
<tr>
<td>Coiflet</td>
<td>6.39</td>
<td>93.61</td>
<td>0.51</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Table 1. Performance comparison of 3 wavelet families for a LOS channel

4 SIMULATION RESULTS AND COMPARISON MODEL
Our model was tested on LOS and NLOS discrete CIR vectors, respectively composed of 1024 and 2048 samples. In each case, the CIR was decomposed on a symmetric wavelet packet family leading to wavelet packet coefficients. The optimal threshold levels were then applied to compress these coefficients. The model performance obtained are summarized in table 2. Despite the high CR obtained, the MSE stays below 10 % in each of the two UWB channel scenarios. This is clearly observed from fig. 3a and 3b which illustrate the initial and reconstructed CIR of LOS and NLOS UWB channels. The reconstructed CIR preserves main
clusters (each cluster combines several multipath responses that are close in time following a decaying law) which contain the greatest part of CIR energy.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Optimal threshold value</th>
<th>MSE (%)</th>
<th>CR (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOS</td>
<td>1.37</td>
<td>6.37</td>
<td>92.85</td>
</tr>
<tr>
<td>NLOS</td>
<td>0.95</td>
<td>5.51</td>
<td>94.59</td>
</tr>
</tbody>
</table>

Table 2. Summary of simulation results using symmetric wavelets and optimal threshold values

![Initial and reconstructed CIR profile for a LOS channel](image)

**Fig 3a.** Initial and reconstructed CIR profile for a LOS channel
Fig 3b. Initial and reconstructed CIR profile for an NLOS channel

In order to have a reference model and to be able to compare the above results, we consider a UWB channel model based on Karhunen-Loève (KL) expansion theorem [5]. This model is constructed by the empirical estimation of the channel correlation matrix using the whole CIR profiles present in our database. Then the signal is projected by scalar product on the eigenvectors of the channel correlation matrix. This projection leads to coefficients which are compressed by hard thresholding. As shown in fig. 4, we iterated the above projections with different threshold levels ($\varepsilon$) and obtained the MSE and coefficient’s Entropy as a function of $\varepsilon$. 
Fig 4. MSE and Entropy of the KL model as a function of threshold, LOS channel

The performance of KL model is summarized in table 3. It is clear that this model provides better results once compared with our proposed model. However the main drawback of the KL model is that its parameters are dependent on channel statistics. Indeed, a receiver using this model for channel estimation will need to have an \textit{a priori} on channel statistics.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Optimal threshold value</th>
<th>MSE (%)</th>
<th>CR (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOS</td>
<td>1.35</td>
<td>5.52</td>
<td>94.48</td>
</tr>
<tr>
<td>NLOS</td>
<td>1.43</td>
<td>3.66</td>
<td>96.34</td>
</tr>
</tbody>
</table>

\textbf{Table 3}. MSE and CR of the KL model for LOS and NLOS channels

The wavelet packet based channel model proposed in this paper has the advantage to represent the channel with few parameters without needing any \textit{a priori} on channel statistics. By comparing table 2 and table 3, we conclude that despite the deterministic nature of our proposed model, its performances are close to the KL model.

It was shown that symmetric wavelet packets are more appropriate for representing UWB CIR in terms of number of wavelet basis vectors to use. According to simulation results, in the case of a
LOS channel, only 71 wavelet basis vectors over 1024 initial vectors can represent the CIR with a reconstruction error equal to 6.37 %. In the NLOS case, the set of vectors useful to represent the CIR contains 103 vectors which are 5 % of the total available 2048 wavelet basis vectors and the MSE after reconstruction is 5.51 %.

5 CONCLUSION
The channel model presented in this paper was based on WPT of the CIR. The main goal of this work was to find a channel model with few parameters and independent of channel’s statistics, leading to high CR and low MSE after compression. It has been shown that the reconstructed CIR achieved a satisfying degree of accuracy, once compared with the KL model. The model performance have also shown to be robust with respect to the CIR small scale variations. Future work will use the proposed channel model in semi-blind UWB channel estimation and multicarrier transmission systems. This model also finds several applications in channel simulation algorithms.

References