In[1]:= `T = {{Exp[β * (J)], Exp[-β * J]}, {Exp[-β * J], Exp[β * (J)]}}`

Out[1]= $\left\{\left\{e^{J\beta}, e^{-J\beta}\right\}, \left\{e^{-J\beta}, e^{J\beta}\right\}\right\}$

In[3]:= `Eigenvalues[T]`

Out[3]= $\left\{e^{-J\beta}\left(-1+e^{2J\beta}\right), e^{-J\beta}\left(1+e^{2J\beta}\right)\right\}$

In[4]:= `Eigenvectors[T]`

Out[4]= {{-1, 1}, {1, 1}}

In[5]:= `U = Transpose[Eigenvectors[T]]`

Out[5]= {{-1, 1}, {1, 1}}

In[6]:= `Tdi = Inverse[U].T.U // FullSimplify`

Out[6]= {{2 Sinh[J β], 0}, {0, 2 Cosh[J β]}}

In[7]:= `Tr[Tdi] // FullSimplify`

Out[7]= $2\, e^{J\beta}$

In[8]:= `Tr[MatrixPower[Tdi, n]] // FullSimplify`

Out[8]= $2^n \left(\mathrm{Cosh}[J\beta]^n + \mathrm{Sinh}[J\beta]^n\right)$

In[9]:= `S = {{1, 0}, {0, -1}}`

Out[9]= {{1, 0}, {0, -1}}

In[10]:= `Tr[S.U.MatrixPower[Tdi, n]. Inverse[U]] // FullSimplify`

Out[10]= 0