

In[24]:= **T = { {Exp[β * (J + H)], Exp[-β * J]}, {Exp[-β * J], Exp[β * (J - H)]} }**

Out[24]= $\left\{ \left\{ e^{(H+J)\beta}, e^{-J\beta} \right\}, \left\{ e^{-J\beta}, e^{-(H+J)\beta} \right\} \right\}$

In[25]:= **Eigenvalues[T]**

Out[25]= $\left\{ \frac{1}{2} e^{-J\beta} \left(e^{J\beta + (-H+J)\beta} + e^{J\beta + (H+J)\beta} - \sqrt{-4(-1 + e^{4J\beta}) + (-e^{J\beta + (-H+J)\beta} - e^{J\beta + (H+J)\beta})^2} \right), \right.$
 $\left. \frac{1}{2} e^{-J\beta} \left(e^{J\beta + (-H+J)\beta} + e^{J\beta + (H+J)\beta} + \sqrt{-4(-1 + e^{4J\beta}) + (-e^{J\beta + (-H+J)\beta} - e^{J\beta + (H+J)\beta})^2} \right) \right\}$

In[26]:= **Eigenvectors[T]**

Out[26]= $\left\{ \left\{ \frac{1}{2} \left(-e^{J\beta + (-H+J)\beta} + e^{J\beta + (H+J)\beta} - \sqrt{-4(-1 + e^{4J\beta}) + (-e^{J\beta + (-H+J)\beta} - e^{J\beta + (H+J)\beta})^2} \right), 1 \right\}, \right.$
 $\left. \left\{ \frac{1}{2} \left(-e^{J\beta + (-H+J)\beta} + e^{J\beta + (H+J)\beta} + \sqrt{-4(-1 + e^{4J\beta}) + (-e^{J\beta + (-H+J)\beta} - e^{J\beta + (H+J)\beta})^2} \right), 1 \right\} \right\}$

In[27]:= **U = Transpose[Eigenvectors[T]]**

Out[27]= $\left\{ \left\{ \frac{1}{2} \left(-e^{J\beta + (-H+J)\beta} + e^{J\beta + (H+J)\beta} - \sqrt{-4(-1 + e^{4J\beta}) + (-e^{J\beta + (-H+J)\beta} - e^{J\beta + (H+J)\beta})^2} \right), \right.$
 $\left. \frac{1}{2} \left(-e^{J\beta + (-H+J)\beta} + e^{J\beta + (H+J)\beta} + \sqrt{-4(-1 + e^{4J\beta}) + (-e^{J\beta + (-H+J)\beta} - e^{J\beta + (H+J)\beta})^2} \right) \right\}, \{1, 1\} \right\}$

In[28]:= **Tdi = Inverse[U].T.U // FullSimplify**

Out[28]= $\left\{ \left\{ - \left(e^{J\beta} \left(\text{Cosh}[2(H-J)\beta] + 2 \text{Cosh}[2J\beta] + \right. \right. \right.$
 $\left. \left. \text{Cosh}[2(H+J)\beta] - 4 \text{Cosh}[H\beta] \sqrt{1 + e^{4J\beta} \text{Sinh}[H\beta]^2} - \text{Sinh}[2(H-J)\beta] - \right. \right.$
 $\left. \left. 6 \text{Sinh}[2J\beta] + \text{Sinh}[2(H+J)\beta] \right) \right) / \left(4 \sqrt{1 + e^{4J\beta} \text{Sinh}[H\beta]^2} \right), 0 \right\},$
 $\left\{ 0, \left(e^{J\beta} \left(\text{Cosh}[2(H-J)\beta] + 2 \text{Cosh}[2J\beta] + \text{Cosh}[2(H+J)\beta] + \right. \right. \right.$
 $\left. \left. 4 \text{Cosh}[H\beta] \sqrt{1 + e^{4J\beta} \text{Sinh}[H\beta]^2} - \text{Sinh}[2(H-J)\beta] - \right. \right.$
 $\left. \left. 6 \text{Sinh}[2J\beta] + \text{Sinh}[2(H+J)\beta] \right) \right) / \left(4 \sqrt{1 + e^{4J\beta} \text{Sinh}[H\beta]^2} \right) \right\} \right\}$

In[30]:= **Tr[Tdi] // FullSimplify**

Out[30]= $2 e^{J\beta} \text{Cosh}[H\beta]$

In[31]:= **Tr[MatrixPower[Tdi, n]] // FullSimplify**

Out[31]= $\left(e^{J\beta} \text{Cosh}[H\beta] - e^{-J\beta} \sqrt{1 + e^{4J\beta} \text{Sinh}[H\beta]^2} \right)^n + \left(e^{J\beta} \text{Cosh}[H\beta] + e^{-J\beta} \sqrt{1 + e^{4J\beta} \text{Sinh}[H\beta]^2} \right)^n$

In[32]= **S = {{1, 0}, {0, -1}}**

Out[32]= {{1, 0}, {0, -1}}

In[33]= **Tr[S.U.MatrixPower[Tdi, n].Inverse[U]] // FullSimplify**

$$\text{Out[33]= } -\frac{1}{\sqrt{1 + e^{4J\beta} \text{Sinh}[H\beta]^2}} e^{2J\beta} \text{Sinh}[H\beta] \left(\left(e^{J\beta} \text{Cosh}[H\beta] - e^{-J\beta} \sqrt{1 + e^{4J\beta} \text{Sinh}[H\beta]^2} \right)^n - \left(e^{J\beta} \text{Cosh}[H\beta] + e^{-J\beta} \sqrt{1 + e^{4J\beta} \text{Sinh}[H\beta]^2} \right)^n \right)$$

$$\text{In[27]= } \text{Limit} \left[-\frac{1}{\sqrt{1 + e^{4J\beta} \text{Sinh}[H\beta]^2}} e^{2J\beta} \text{Sinh}[H\beta] \left(\left(e^{J\beta} \text{Cosh}[H\beta] - e^{-J\beta} \sqrt{1 + e^{4J\beta} \text{Sinh}[H\beta]^2} \right)^n - \left(e^{J\beta} \text{Cosh}[H\beta] + e^{-J\beta} \sqrt{1 + e^{4J\beta} \text{Sinh}[H\beta]^2} \right)^n \right), J \rightarrow 2 \right]$$

$$\text{Out[27]= } -\frac{1}{\sqrt{1 + e^{8\beta} \text{Sinh}[H\beta]^2}} e^{4\beta} \text{Sinh}[H\beta] \left(\left(e^{2\beta} \text{Cosh}[H\beta] - e^{-2\beta} \sqrt{1 + e^{8\beta} \text{Sinh}[H\beta]^2} \right)^n - \left(e^{2\beta} \text{Cosh}[H\beta] + e^{-2\beta} \sqrt{1 + e^{8\beta} \text{Sinh}[H\beta]^2} \right)^n \right)$$

$$\text{In[28]= } \text{Limit} \left[-\frac{1}{\sqrt{1 + e^{8\beta} \text{Sinh}[H\beta]^2}} e^{4\beta} \text{Sinh}[H\beta] \left(\left(e^{2\beta} \text{Cosh}[H\beta] - e^{-2\beta} \sqrt{1 + e^{8\beta} \text{Sinh}[H\beta]^2} \right)^n - \left(e^{2\beta} \text{Cosh}[H\beta] + e^{-2\beta} \sqrt{1 + e^{8\beta} \text{Sinh}[H\beta]^2} \right)^n \right), n \rightarrow 100 \right]$$

$$\text{Out[28]= } -\frac{1}{\sqrt{1 + e^{8\beta} \text{Sinh}[H\beta]^2}} e^{4\beta} \text{Sinh}[H\beta] \left(\left(e^{2\beta} \text{Cosh}[H\beta] - e^{-2\beta} \sqrt{1 + e^{8\beta} \text{Sinh}[H\beta]^2} \right)^{100} - \left(e^{2\beta} \text{Cosh}[H\beta] + e^{-2\beta} \sqrt{1 + e^{8\beta} \text{Sinh}[H\beta]^2} \right)^{100} \right)$$

$$\text{In[29]= } \text{Limit} \left[-\frac{1}{\sqrt{1 + e^{8\beta} \text{Sinh}[H\beta]^2}} e^{4\beta} \text{Sinh}[H\beta] \left(\left(e^{2\beta} \text{Cosh}[H\beta] - e^{-2\beta} \sqrt{1 + e^{8\beta} \text{Sinh}[H\beta]^2} \right)^{100} - \left(e^{2\beta} \text{Cosh}[H\beta] + e^{-2\beta} \sqrt{1 + e^{8\beta} \text{Sinh}[H\beta]^2} \right)^{100} \right), \beta \rightarrow 0 \right]$$

Out[29]= 0

$$\text{In[34]= } \text{Limit} \left[\frac{1}{\sqrt{1 + e^{4J\beta} \text{Sinh}[H\beta]^2}} e^{2J\beta} \text{Sinh}[H\beta] \left(\left(e^{J\beta} \text{Cosh}[H\beta] - e^{-J\beta} \sqrt{1 + e^{4J\beta} \text{Sinh}[H\beta]^2} \right)^n - \left(e^{J\beta} \text{Cosh}[H\beta] + e^{-J\beta} \sqrt{1 + e^{4J\beta} \text{Sinh}[H\beta]^2} \right)^n \right), H \rightarrow 0 \right]$$

Out[34]= 0