In the name of God

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STOCHASTIC PROCESSES

Exercise Set 8

(Date Due: 1400/02/20)

1. Show that for stationary time series, $\alpha(t)$, power spectrum is given by $S(\omega)=\tilde{\alpha}(\omega)\tilde{\alpha}(\omega')^T\delta_D(\omega-\omega')$.

2. calculate $\langle n_{up}(\vartheta)\rangle$ for a non-Gaussian random field in (1+1)-dimension up to $\mathcal{O}(\sigma_0^2)$.

3. Suppose that for an isotropic stochastic field in D-dimension $\alpha \equiv \frac{f}{\sigma_0}$, $\vec{\eta}=\vec{\nabla}\alpha$ and $\xi=\nabla^2\alpha$ Show that:

$$\langle \alpha^2\rangle = 1$$

$$\langle \eta_1^2\rangle = \langle \eta_2^2\rangle = \langle \eta_3^2\rangle = \frac{1}{D}\frac{\sigma_1^2}{\sigma_0^2}$$

$$\tilde{\xi}_{ij} \equiv \xi_{ij} + \frac{1}{D}\frac{\sigma_1^2}{\sigma_0^2}$$

$$\langle \tilde{\xi}_{11}^2\rangle = \langle \tilde{\xi}_{22}^2\rangle = \langle \tilde{\xi}_{33}^2\rangle = \frac{3}{D(D+2)}\frac{\sigma_2^2}{\sigma_0^2}\left(1-\frac{D+2}{3D}\gamma^2\right)$$

$$\langle \tilde{\xi}_{11}\tilde{\xi}_{22}\rangle = \langle \tilde{\xi}_{11}\tilde{\xi}_{33}\rangle = \langle \tilde{\xi}_{22}\tilde{\xi}_{33}\rangle = \frac{1}{D(D+2)}\frac{\sigma_2^2}{\sigma_0^2}\left(1-\frac{D+2}{D}\gamma^2\right)$$

$$\langle \tilde{\xi}_{12}^2\rangle = \langle \tilde{\xi}_{13}^2\rangle = \langle \tilde{\xi}_{23}^2\rangle = \frac{1}{D(D+2)}\frac{\sigma_2^2}{\sigma_0^2}$$

4. Compute $\langle |\eta_i|\rangle$ for 1 and 2 and 3 dimensions.

Good luck, Movahed
