In the name of God

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STOCHASTIC PROCESSES

Exercise Set 5

(Date Due: 1398/02/24)

1. Bias definition: The relation between Unweighted TPCF and weighted TPCF can be considered as:

$$\Psi_{fg}(R) = \mathcal{B}_{fg}C(R)$$

   (a) For Up-crossing feature determine $\mathcal{B}$ for 1+1-D stochastic field.

   (b) For a sharp clipping feature, we have $f \equiv A\Theta(\alpha - \vartheta)$. Here A is a normalization coefficient. The average value of sharp clipping in a Gaussian random field can be written by:

$$\langle f(\alpha)\rangle = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} A\Theta(\alpha - \vartheta)e^{-\alpha^2/2}d\alpha$$

   Now, we expand $\Theta(\alpha - \vartheta)$ in terms of Hermite polynomials defined by $H_n = e^{x^2/2}(d/dx)^n e^{-x^2/2}$ with $\langle H_n(x)H_m(x)\rangle \delta_{nm} m!$. Therefore we have

$$A\Theta(\alpha) = \sum_{k=0}^{\infty} \frac{J_k}{k!} H_k(\alpha)$$

   Show that

$$J_k = \frac{\vartheta H_{k-1}(\vartheta)}{\vartheta\sqrt{\pi/2}\exp(\vartheta^2/2)\mathrm{erfc}(\vartheta/\sqrt{2})}$$

   (c) Show $\lim_{\vartheta \to \infty} J_k = \vartheta^k$

   (d) Show $\langle \Theta(\alpha(r))\Theta(\alpha(r'))\rangle = 1 + \Psi(R) = \sum_{k=0}^{\infty} \frac{J_k^2}{k!} C_{\alpha\alpha}^k(R)$, where $R = |r - r'|$.

   (e) For $R \to \infty$ show $\langle \Theta(\alpha(r))\Theta(\alpha(r'))\rangle = 1 + \Psi(R) \sim J_1^2 C_{\alpha\alpha}(R)$. Here $J_1 = \vartheta^2$.

2. Compute the scale-independent bias of peak for 1+2-D.

Good luck, Movahed