In the name of God

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STOCHASTIC PROCESSES

Exercise Set 1

(Date Due: 1399/12/10)

1. Error analysis and propagation: Using the input file, write a proper program file to do following tasks:

A : Read input data file which contains more than 10^6 one-column data. and spilt it to 100 input files.
B : Making directories and send each data sets to corresponding directory.
C : Compute mean, variance and mean standard deviation of each data sets. And write them in a file which contains the label of data, mean, standard deviation and mean standard deviation. Finally plot them.
D : Compute $p_i(x)$ as a function of x for each sets. To this end, use $W(x, q)$ for $q = 0$ and $q = 1$.
E : Smooth the computed $p_i(x)$ in the previous question with a Gaussian kernel with a typical variance and finally plot them.
F : Compute $\sigma_m(p(x))$. Plot $p(x)$ versus x and show its error-bar for 5 sets of data.

2. Convolution theorem: For a kernel estimation it is useful to use $F(x) = \mathcal{K} \bigotimes f = \int \mathcal{K}(x - x')f(x')dx'$.
A : Write convolution in the Fourier space and explain the motivation behind such transformation.
B : To find the output of the system with impulse response $h(t) = e^{-2t}, \quad t > 0$ to the input $f(t) = 0$ for $t < 0$, $f(t) = 1$, for $0 \geq t \geq 1$ and $f(t) = 0$ for $t > 1$. Use the convolution integral form as $y(t) = \int_{-\infty}^{+\infty} f(\lambda).h(t - \lambda)d\lambda$ and explain your results.

3. Different Probability density functions:

A : For A binomial distribution, compute $\langle k \rangle$, $\langle (k - \langle k \rangle)^2 \rangle$, $\langle (k - \langle k \rangle)^3 \rangle$ and show $P(k)$ for binomial is normalized.
B : For A Poisson distribution, compute $\langle k \rangle$, $\langle (k - \langle k \rangle)^2 \rangle$, $\langle (k - \langle k \rangle)^3 \rangle$ and show $P(k)$ for binomial is normalized. Also show:

$$P_{poisson}(k) = \lim_{N \to \infty} P_{binomial}(k)$$

C : Show $P_{Gaussian}(k) = \lim_{\lambda \to \infty} P_{poisson}(k)$

Good luck, Movahed