In the name of God

Department of Physics
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STOCHASTIC PROCESSES

Exercise Set 13

(Date Due: 1400/03/23)

1. Application of Fokker-Planck equation to brownian motion in a Harmonic Potential:
Brownian motion in a harmonic potential with an external force $F(t)$ is described by the Langevin equation as:

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 = \frac{F(t)}{m} + \eta(t)$$

here $\langle\eta(t)\rangle = 0$ and $\langle\eta(t)\eta(t')\rangle = \delta(t - t')$
a) Considering linearity of the system and based on physical meaning of weighted TPCF, the response function is directly related to $\langle x(\tau)x(0)\rangle$ which is represented by $R_x^{\text{Linear}}(t)$. Now compute $R_x^{\text{Linear}}(t)$.
b) Calculate the response of the velocity $R_v^{\text{Linear}}(t)$.
c) Calculate the power spectrum of velocity and position.
d) Considering $\chi_x(\omega) = \int_0^\infty dte^{-i\omega\tau} R_x^{\text{Linear}}(\tau)$ and $\chi_v(\omega) = \int_0^\infty dte^{-i\omega\tau} R_v^{\text{Linear}}(\tau)$, show that $\chi_v(\omega) = i\omega\chi_x(\omega)$.
e) What about stationary regime? (Hint: $\gamma\tau \to \infty$ and $F(t) = 0$)
f) What about $\gamma = 0$ and $F(t) = 0$?

2. Show that generalized drift coefficient transform as contravariant vector if we define:

$$\bar{D}^i \equiv D^i - \sqrt{Det}\frac{\partial}{\partial x^j}\frac{D^{ij}}{\sqrt{Det}}$$

3. Show that covariant derivative of probability current is scalar

$$\bar{S}^i_{;i} \equiv \sqrt{Det}\frac{\partial}{\partial x^i}\frac{\bar{S}^i}{\sqrt{Det}}$$

Good luck, Movahed