

In the name of God

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STOCHASTIC PROCESSES

Exercise Set 12

(Date Due: 1400/03/20)

1. Using Ito's and Stratonovich's definitions show that:

$$D_I^{(1)} = h(v, t)$$

$$D_I^{(2)} = g^2(v, t)$$

$$D_S^{(1)} = h(v, t) + g'(v, t)g(v, t)$$

$$D_S^{(2)} = g(v, t)^2$$

Write non-linear Langevin equation for each approaches .

2. Markov process:

(a) Show that correlation function is given by:

$$C_x(t) = \langle x e^{t|\mathcal{L}_{KM}^+} x \rangle$$

where

$$\mathcal{L}_{KM}^+ = \sum_n D^{(n)}(x) \left(\frac{\partial}{\partial x} \right)^n$$

(b) For $\dot{x}(t) = -ax^3(t) + \eta(t)$ determine the one-point probability density function, $p(x)$. Suppose that $D^{(n)} = 0$ for $n \geq 3$.

(c) Using calculated PDF in the above part, determine correlation function. Suppose that the noise behaves as $\langle \eta(t)\eta(t') \rangle = 2q\delta(t - t')$.

Good luck, Movahed
