In the name of God

Department of Physics
Shahid Beheshti University

STOCHASTIC PROCESSES

Exercise Set 10

(Due Date: 1401/03/27)

1. According to following definition:

$$\int_{-\infty}^{+\infty} x^n \exp[-(x-\beta)^2]dx = (2i)^{-n}\sqrt{\pi}H_n(i\beta)$$

where H_n is the Hermite polynomial, show that conditional moment reads as:

$$M_n(x',t,\tau) = \left[-i\sqrt{D^{(2)}(x',t)\tau}\right]^n H_n\left\{\frac{1}{2}iD^{(1)}(x',t)\sqrt{\tau/D^{(2)}(x',t)}\right\}$$

also show that above equation causes to correct function for $D^{(n)}$.

2. Using Green's function approach show that:

$$\dot{G}_{ij} + \xi_{ik}G_{kj} = 0$$

3. According to forward solution, and suppose that $D^{(4)}(x,t) = 0$, show that:

$$p(x,t+\tau|x',t) = \left[1 - \frac{\partial}{\partial x}D^{(1)}(x,t)\tau + \frac{\partial^2}{\partial x^2}D^{(2)}(x,t)\tau\right]\delta(x-x')$$

has the following solutions:

(a) $p(x,t+\tau|x',t) = \frac{1}{2\sqrt{\pi D^{(2)}(x',t)\tau}} \exp\left(-\frac{[x-x'-D^{(1)}(x',t)\tau]^2}{4D^{(2)}(x',t)\tau}\right)$

(b) $p(x,t+\tau|x',t) = \frac{1}{2\sqrt{\pi D^{(2)}(x,t)\tau}} \exp\left(-\frac{\partial}{\partial x}D^{(1)}(x,t)\tau + \frac{\partial^2}{\partial x^2}D^{(2)}(x,t)\tau - \frac{[x-x'-(D^{(1)}(x,t)-2\frac{\partial}{\partial x}D^{(2)}(x,t))\tau]^2}{4D^{(2)}(x,t)\tau}\right)$

4. Path integral solution: According to Markovian property show that:
$p(x,t) = \lim_{N\to\infty} \int ... \int \Pi_{i=0}^{N-1}\left\{\frac{dx_i}{\sqrt{4\pi D^{(2)}(x_i,t_i)}}\right\} \times \exp\left(-\sum_{i=0}^{N-1}\frac{[x_{i+1}-x_i-D^{(1)}(x_i,t_i)\tau]^2}{4D^{(2)}(x_i,t_i)\tau}\right)p(x_0,t_0)$
The summation in exponential can be written by *Generalized Onsager-Machlup* function for discrete case.

5. Show that the forward and backward Kramers-Moyal expansion are equivalent.

6. According to:

$$p(x,t+\tau|x',t)p(x',t) = p(x+\Delta-\Delta,t+\tau|x-\Delta,t)p(x-\Delta,t)$$

where $x' = x - \Delta$, derive the forward Kramers-Moyal expansion. (hint: this is called the third approach of expansion derivation mentioned in the class.

7. Exercise 3.4 and 3.5 of book "Analysis and Data-Based Reconstruction of Complex Nonlinear Dynamical Systems Using the Methods of Stochastic Processes", written by M. Reza Rahimi Tabar.

Good luck, Movahed