Stochastic991219

بسم الله الرحمن الرحیم

جلسه ۹۹/۱۲/۱۹

با استفاده از شکل

$$P(A) = \hat{\mathcal{L}}\, P_G(A)$$

$$\langle f \rangle = \int d^N A\, f(A)\, P(A)$$

$$= \int d^N A\, f(A)\, \hat{\mathcal{L}}\, P_G(A)$$

$$= \int d^N A\, P_G(A)\, \hat{\mathcal{L}}^{+} f(A)$$

$$= \langle \mathcal{L}^{+} f(A) \rangle_G$$

$$\boxed{\hat{K}^{(n)} \equiv \frac{K^{(n)}}{\sigma_0^{\,n-2}}}\ , \quad \sigma_0^2 \equiv K^{(2)} = K_2$$

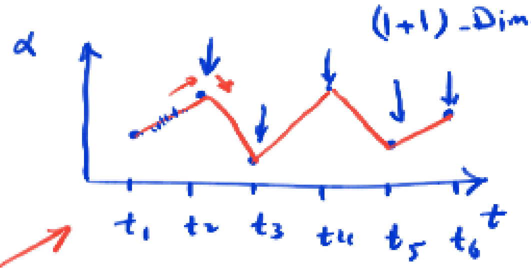
$$A : \{A_\mu\} = \{\alpha, \partial_1\alpha, \partial_2\alpha, \ldots, \partial_{11}\alpha, \ldots\}$$

$\alpha \equiv$ میدان تصادفی

→ مشتق پذیر ←

↑

(1+1)_Dimension



Recorded Data → Digitalized

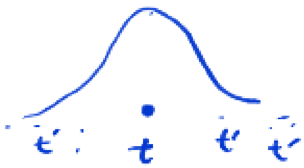
Smoothed stochastic field

مثلا

$$\{\alpha(t)\} \longrightarrow \bar{\alpha}(t) = \int dt' \, \underbrace{\widetilde{W(t-t')}}_{I} \, \alpha(t')$$

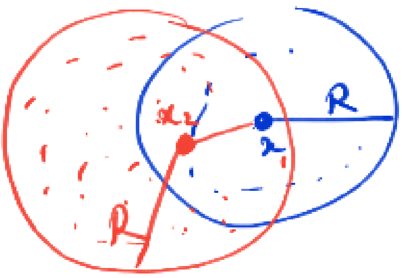
Kernel (Window function)

$W \equiv$ Gaussian



$$\alpha(x) \longrightarrow \bar{\alpha}(x) = \int dx' \, \underbrace{W(x-x')}_{R} \, \alpha(x')$$

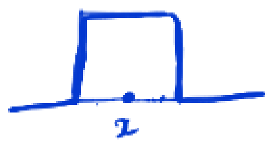
$W(x-x')$



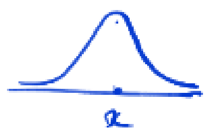
3D.

step-function

$$W_R(x) = \frac{3}{4\pi R^3} \cdot \Theta(R-x) \longrightarrow$$



$$W_R(x) = \frac{1}{(2\pi)^{3/2} R^3} \, e^{-\frac{x^2}{2R^2}} \longrightarrow$$



$\alpha(\vec{r}) \rightarrow$ میدان تصادفی

$$A_p : \{\alpha, \partial_1 \alpha, \partial_2 \alpha, \partial_{11} \alpha\}$$

$$: \{\alpha(\vec{r}), \eta_1, \eta_2, \xi_{11}, \dots\}$$

$$\left\{ \begin{array}{l} \vec{\eta} = \vec{\nabla} \alpha \\ \xi_{ij} = \partial_i \partial_j \alpha \end{array} \right\}$$

$$\alpha(\vec{r}) \rightarrow \quad \bar{\alpha} = \frac{\alpha - \langle \alpha \rangle}{}$$

σ_0

$$A_\mu \to \bar{A}_\mu : \{ \bar{\alpha}, \dots \} \qquad \sigma_0^2 \equiv K_\alpha^{(2)} = \langle \alpha^2 \rangle - \langle \alpha \rangle^2$$

$$\bar{\sigma}_0^2 \; K_{\bar{\alpha}}^{(2)} = \langle \bar{\alpha}^2 \rangle - \langle \bar{\alpha} \rangle^2 = 1$$

$$\bar{A}_\mu \to A_\mu$$

ز بزرگی علامت = بدیم.

$$K_\alpha^{(2)} = 1$$

$$\boxed{e^x = 1 + x + \frac{x^2}{2} + \dots}$$

$$\langle f \rangle = \langle \mathcal{L}^\dagger f \rangle_G \qquad \tilde{K} \sim \mathcal{O}(\sigma_0^{n-2})$$

پس

$$\langle f \rangle = \Big\langle \underbrace{\exp\Big(\sum_{n=3}^{\infty} \frac{(+1)^n}{n!} \Big\{ \sum_{\mu_1=1}^{N} \cdots \sum_{\mu_n=1}^{N} K^{(n)}_{\mu_1 \dots \mu_n} \frac{\partial^n}{\partial A_{\mu_1} \cdots \partial A_{\mu_n}} \Big\} \Big)}_{\text{بسط بدیم}} f \Big\rangle_G$$

exp( )

$$= \Big\langle \Big\{ 1 + \sum_{n=3}^{\infty} \frac{(+1)^n}{n!} \Big\{ \sum_{\mu_1=1}^{N} \cdots \sum_{\mu_n=1}^{N} K^{(n)}_{\mu_1 \dots \mu_n} \frac{\partial^n}{\partial A_{\mu_1} \cdots \partial A_{\mu_n}} \Big\}$$

$$+ \Big(\sum_{n=3}^{\infty} \frac{(+1)^n}{n!} \Big\{ \sum_{\mu_1=1}^{N} \cdots \sum_{\mu_n=1}^{N} K^{(n)}_{\mu_1 \dots \mu_n} \frac{\partial^n}{\partial A_{\mu_1} \cdots \partial A_{\mu_n}} \Big\} \Big)^2 + \dots \Big\}$$

$$f \Big\rangle_G$$

① ↓

$N \quad N \quad N \quad (3)$

$$\langle f\rangle = \langle f\rangle_G + \frac{1}{3!}\sum_{\mu_1=1}\sum_{\mu_2=1}\sum_{\mu_3=1} K_{\mu_1\mu_2\mu_3}\langle f_{,\mu_1\mu_2\mu_3}\rangle_G\,\sigma_0$$

$$+\Big[\frac{1}{4!}\sum_{\mu_1=1}^{N}\sum_{\mu_2=1}^{N}\sum_{\mu_3=1}^{N}\sum_{\mu_4=1}^{N} K^{(4)}_{\mu_1\mu_2\mu_3\mu_4}\langle f_{,\mu_1\mu_2\mu_3\mu_4}\rangle_G$$

$$+\frac{1}{2(3!)^2}\sum_{\mu_1=1}^{N}\sum_{\mu_2=1}^{N}\sum_{\mu_3=1}^{N} K^{(3)}_{\mu_1\mu_2\mu_3}\sum_{\nu_1=1}^{N}\sum_{\nu_2=1}^{N}\sum_{\nu_3=1}^{N} K^{(3)}_{\nu_1\nu_2\nu_3}$$

$$\langle f_{,\mu_1\mu_2\mu_3\nu_1\nu_2\nu_3}\rangle_G\Big]\sigma_0^2 + \mathcal{O}(\sigma_0^3)$$

Exercise $\mathcal{O}(\sigma_0^4) \longrightarrow \langle f\rangle = ?$

$\boxed{N=1}$

Example: $p(x) = ?$ for $\widetilde{A:\{x\}}$

اگر میدان تصادفی نرم شده $\{x\}$ باشد. تابع توزیع پیدا کنید (به صورت اختلالی)

$$\langle f\rangle = \langle \hat{\mathcal{L}}^{+} f\rangle_G \longrightarrow \widetilde{p(x)} = ?$$

یادآوری

$$p(x) = \langle \delta_D(x-x')\rangle_{x'} \longrightarrow \boxed{f \equiv \delta_D(x-x')}$$

$$x \longrightarrow \bar{x} = \frac{x-\langle x\rangle}{\sigma_0} \qquad \bar{x}\equiv\alpha \quad \longrightarrow A:\{\alpha\}$$

$$\sigma_0 \longleftarrow \sigma_0^2 = \langle x^2\rangle - \langle x\rangle^2$$

$$p(x) \longrightarrow p(\nu) = ?$$

$$p_\alpha(\nu) = \left\langle \underbrace{\delta_D(\nu-\alpha)}_{f} \right\rangle_\alpha$$

$$① \quad p_\alpha(\nu) = \left\langle \delta_D(\nu-\alpha) \right\rangle_G + \overbrace{\frac{1}{3!} \sum_{\mu_1=1}^{N=1} \sum_{\mu_2=1}^{1} \sum_{\mu_3=1}^{1} K^{(3)}_{\mu_1\mu_2\mu_3}}^{B} \left\langle \overbrace{\frac{\partial^3 \delta(\alpha-\nu)}{\partial A_{\mu_1} \partial A_{\mu_2} \partial A_{\mu_3}}}^{B} \right\rangle_G \sigma_0 + \dots\dots$$

$$= \int d\alpha \, p_G(\alpha) \, \delta_D(\nu-\alpha) + \dots\dots + \dots$$

$$p_a(\nu) = \frac{1}{\sqrt{2\pi}} e^{-\frac{\nu^2}{2}} + \dots\dots + \dots$$

$$Ⓑ \equiv \frac{\sigma_0}{3!} \sum_{\mu_1=1}^{1} \sum_{\mu_2=1}^{1} \sum_{\mu_3=1}^{1} K^{(3)}_{\mu_1\mu_2\mu_3} \left\langle \frac{\partial^3 \delta}{\partial A_{\mu_1} \partial A_{\mu_2} \partial A_{\mu_3}} \right\rangle_G$$

$$\boxed{Ⓑ = \frac{\sigma_0}{3!} K^{(3)}_{111} \left\langle \frac{\partial^3 \delta_D(\alpha-\nu)}{\partial \alpha^3} \right\rangle_G}$$

$\boxed{N:1}$

$$K^{(3)}_{111} = \langle \alpha\alpha\alpha \rangle$$

$$\left\langle \frac{\partial^3 \delta_D(\alpha-\nu)}{\partial \alpha^3} \right\rangle_G = ?$$

Exercise

$$\left\{\begin{aligned} \left\langle \frac{\partial^K \delta_D(\alpha-\nu)}{\partial \alpha^K} H_n(\alpha) \right\rangle_G &\overset{?}{=} \frac{e^{-\nu^2/2}}{\sqrt{2\pi}} H_{n+K}(\nu) \\ \left\langle \frac{\partial^K \theta(\alpha-\nu)}{\partial \alpha^K} H_n(\alpha) \right\rangle_G &\overset{?}{=} \frac{e^{-\nu^2/2}}{\sqrt{2\pi}} H_{n+K-1}(\nu) \end{aligned}\right\}$$

$$\boxed{\begin{array}{l} \text{Probabilistic Hermit function} \\ H_n(x) \equiv e^{\nu^2/2} \left(-\frac{\partial}{\partial \nu}\right)^n e^{-\nu^2/2} \end{array}}$$

$$\left\langle \frac{\partial^3 \delta_D(\alpha-\nu)}{\partial \alpha^3} \right\rangle_G = \left\langle \frac{\partial^3 \delta_D(\alpha-\nu)}{\partial \alpha^3} H_0 \right\rangle_G$$

$$= \frac{e^{-\nu^2/2}}{\sqrt{2\pi}} H_3(\nu)$$

$$\boxed{\beta = \frac{\sigma_0}{3!} K^{(3)}_{111} \frac{e^{-\nu^2/2}}{\sqrt{2\pi}} H_3(\nu)}$$

σ [illegible]

$$P_\alpha(\nu) = \frac{1}{\sqrt{2\pi}} e^{-\nu^2/2} + \frac{\sigma_0}{3!} \frac{e^{-\nu^2/2}}{\sqrt{2\pi}} K^{(3)}_{111} H_3(\nu) + \cdots$$

$K^{(4)}_{1111}$

$$P_\alpha = \frac{1}{\sqrt{2\pi}} e^{-\nu^2/2} \left[1 + \frac{\sigma_0}{3!} K^{(3)}_{111} H_3(\nu) + \cdots \right]$$

$$P_\alpha(\nu) = \frac{1}{\sqrt{2\pi}} e \left[\cdots \frac{\;}{3!} \cdots 3 \cdots \right]$$

Skewness $= \langle \alpha\alpha\alpha \rangle$ Errors

$$K^{(n)} \sim \mathcal{O}(\sigma_0^{\,n-2}) \qquad \sigma_0 < 1$$

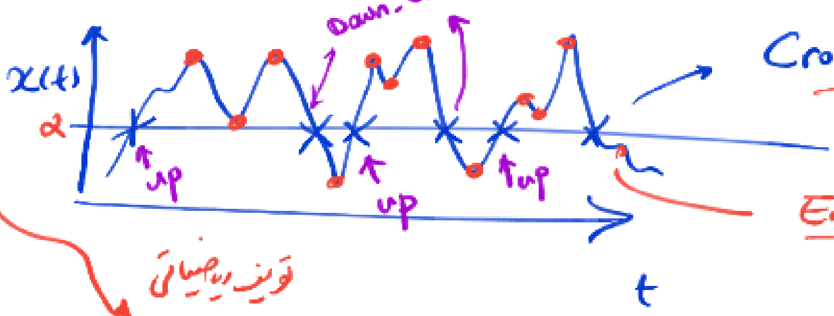
$$f \to \boxed{\delta_D(\alpha - \nu)} \longrightarrow P_\alpha(\nu) = P_\alpha(\nu)\big|_G + \text{Non-Gaussian}$$

P δ_D

خواص هندسی میدان مورد مطالعه بود

$$\langle f \rangle = \langle \text{ویژگی های دلخواه مرتبط به سیستم مورد مطالعه} \rangle$$

$$\langle f \rangle = \langle \text{Conditions Correspond to feature} \rangle_G + \langle \text{Condition Correspond to feature} \rangle_{NON-G}$$



Crossing_Statistic

Extremum Statistic

تعریف ریاضیاتی

$$\langle n_{Peak}(\alpha) \rangle = ?$$ خطای شرایط قله ها

$$\langle n_{Crossing}(\alpha) \rangle = ?$$ خطای شرایط برخورد

$\vdots$

$\{$ Critical Set $\}$

Example Up-Crossing = ?

Peak = ?