Stochastic991217

جلسه ۹۹/۱۲/۱۷

بسم الله الرحمن الرحیم

$$K_1 = \langle A_\mu \rangle = 0$$

$$\begin{cases} n=1 \longrightarrow K_1 = 0 \\ n=2 \longrightarrow K^{(2)} \neq 0 \end{cases}$$

$$Z_A(\vec{\lambda}) = \exp\left(-\frac{1}{2}\lambda^T \cdot K^{(2)} \cdot \lambda\right)$$

$$\boxed{\times \exp\Big[\sum_{n=3} \frac{i^n}{n!} \Big(\underbrace{\sum_{\mu_1=1}^{N} \cdots \sum_{\mu_n=1}^{N}}_{n} \underbrace{K^{(n)}_{\mu_1 \cdots \mu_n}}_{n} \underbrace{\lambda_{\mu_1} \cdots \lambda_{\mu_n}}_{n}\Big)\Big]}$$

Example → تک متغیره

قبل از اینکه جلوتر بریم یک برمی‌گردیم به حالت

$\mu: 1$ یعنی $\boxed{A_\mu : (x)}$

$K_1 = \langle x \rangle_c = \langle x \rangle = 0$ فرض

$$Z_x(\lambda) = \exp\left[-\frac{1}{2}\lambda^2 K^{(2)}\right] \times \exp\left[\frac{i^3}{3!} K^{(3)} \lambda^3 + \frac{i^4}{4!} K^{(4)} \lambda^4 + \cdots\right]$$

for Gaussian field (process) if $K_1 = 0 \longrightarrow \{K_n = 0 \text{ for } n \geq 3\}$

واریانس $\sigma_x^2 \equiv K_2 = \langle x^2 \rangle_c = \langle x^2 \rangle - \langle x \rangle^2 \longleftarrow K_2 \neq 0$ یعنی فقط

$= \langle x^2 \rangle$

Gaussian

$$Z_x(\lambda) = e^{-\frac{\lambda^2 K^{(2)}}{2}} \longleftarrow$$ پس یعنی گوسی بوده است

$$P(x) = \frac{1}{2\pi} \int d\lambda \, e^{-i\lambda x} Z_x(\lambda)$$

$$p(x) = \frac{1}{2\pi} \int d\lambda \, \underbrace{e^{-i\lambda x} e^{-\frac{\lambda^2 K_2}{2}}}$$ → $\boxed{\text{G\_function}}$

$$-i\lambda x - \frac{\lambda^2 K_2}{2} + \frac{x^2}{K_{\ldots}} - \frac{x^2}{\ldots}$$

$$= e \quad 1 \quad K_2 \quad K_2$$

$$\left(+\frac{i\lambda K_2^{1/2}}{2^{1/2}} + \frac{x}{2^{1/2}K_2^{1/2}}\right)^2$$

$$\frac{x^2}{K_2} - \frac{\lambda^2 K_2}{2} + \cancel{2}\, i\lambda K_2^{1/2} \frac{x}{\cancel{x}\, K_2^{1/2}}$$

$$p(x) = \frac{1}{\sqrt{2\pi K_2}}\, e^{-\frac{x^2}{2K_2}} \quad \left\{\begin{matrix}\text{Gauss}\\ i\lambda K_1 \neq 0\end{matrix}\right\}$$

$$(x-\langle x\rangle)^2$$

$$\boxed{\left.p(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}}\, e^{-\frac{x^2}{2\sigma_x^2}}\right\} \equiv \mathcal{N}(0, \sigma_x) \quad \mathcal{N}(\langle x\rangle, \sigma_x)} \rightarrow \text{Gaussian process}$$

$$A_\mu, \quad \underbrace{\mu = 1 \ldots N}$$

در حالت کلی

$$K_1 = \langle A_\mu\rangle = 0$$

حالت کلی را محدود می کند

$$Z_A(\vec{\lambda}) = $$ قبلاً نوشتیم $$\underbrace{-i\lambda^\mu A_\mu}$$

$$-i\vec{\lambda}\cdot\vec{A}$$

$$P(A_\mu) = \frac{1}{(2\pi)^N}\int d^N\lambda\, Z_A(\vec{\lambda})\, e^{-i\vec{\lambda}\cdot\vec{A}}$$

N-Joint PDF

$$= \frac{1}{(2\pi)^N}\int_{-\infty}^{+\infty} d^N\lambda\, e^{-\frac{1}{2}\lambda^T\cdot K_2\cdot\lambda - i\vec{\lambda}\cdot\vec{A}}$$

$$\lambda_1 A_1 + \lambda_2 A_2 - - -$$

$$\sum_{n=3}^{\infty}\frac{i^n}{n!}\left\{\sum_{\mu_1=1}^{N}\cdots\sum_{\mu_n=1}^{N} K^{(n)}_{\mu_1\ldots\mu_n}\,\lambda_{\mu_1}\ldots\lambda_{\mu_n}\right\}$$

$\times\, e$

$$\boxed{\lambda_\mu \rightarrow i\frac{\partial}{\partial A_\mu}}$$

$$\boxed{P(A_\mu) = \exp\left[\sum_{n=3}^{\infty} \frac{i^{2n}}{n!}\left(\sum_{\mu_1=1}^{N}\cdots\sum_{\mu_n=1}^{N} K^{n}_{\mu_1\cdots\mu_n}\frac{\partial^n}{\partial A_{\mu_1}\cdots\partial A_{\mu_n}}\right]\times \frac{1}{(2\pi)^N}\int d^N\lambda\; e^{-\frac{1}{2}\vec{\lambda}^T\cdot K_2\cdot\lambda - i\vec{\lambda}\cdot\vec{A}}}$$

Recall that $\ast$ $P_G(A_\mu) = \frac{1}{(2\pi)^{N/2}}\int d^N\lambda\; e^{-i\vec{\lambda}\cdot\vec{A} - \frac{\lambda^T\cdot K^{(2)}\cdot\lambda}{2}}$ $\ast$

$\boxed{\text{N-Joint Gaussian PDF}}$

به طور سیستماتیک هر تابع توزیع دلخواهی به صورت زیر می‌توان ایجاد کرد یا استخراج کرد.

$$\ast\;\boxed{\underbrace{P(\vec{A})}_{\text{N-Joint PDF}} = P(A_\mu) = \hat{\mathcal{L}}\;\underbrace{P_G(A_\mu)}_{\text{N-Joint Gaussian PDF}}}\;\ast$$

بله

$$\hat{\mathcal{L}} \equiv \exp\left[\sum_{n=3}^{\infty}\frac{(-1)^n}{n!}\left\{\sum_{\mu_1=1}^{N}\cdots\sum_{\mu_n=1}^{N} K^{(n)}_{\mu_1\cdots\mu_n}\frac{\partial^n}{\partial A_{\mu_1}\cdots\partial A_{\mu_n}}\right\}\right]$$

$n=1 - K_1 = 0$

$n=2 - K_2 \rightarrow P_G(A_\mu)$

$n \geq 3$

if $p(A_\mu) \equiv$ Gaussian $\to \kappa_n = 0 \quad n>2$

$$\mathcal{L} = 1 \to p(A_\mu) = 1\, p_G(A_\mu)$$

"Perturbative Expansion" $\quad p(A_\mu) = \hat{\mathcal{L}}\, p_G(A_\mu)$

همچنین اینطوری که لزوماً جملات با n بزرگتر از جملات
n کوچکتر باشد

$$\left\{ \begin{array}{l} n \nearrow \\ \kappa_n \searrow \end{array} \right\}$$

$$p(A_\mu) = \hat{\mathcal{L}}\, p_G(A_\mu)$$

$$\underbrace{p(A_\mu)}_{\text{مختل}} = \boxed{\hat{\mathcal{L}}_1\, p_G(A_\mu) + \hat{\mathcal{L}}_2\, p_G(A_\mu) + \hat{\mathcal{L}}_3\, p_G(A_\mu) + \cdots + \cdots\cdots\ \hat{\mathcal{L}}_M\, p_G(A_\mu) + \cdots}$$

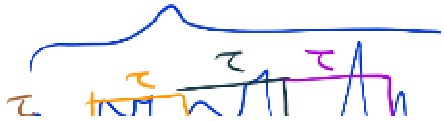
همیشه اتفاق نمی‌افتد $\longrightarrow \quad \hat{\mathcal{L}}_1 > \hat{\mathcal{L}}_2 > \hat{\mathcal{L}}_3 \cdots > \mathcal{L}_M \cdots$

و هرچقدر که $p(A_\mu)$ از حالت گوسی بیشتر منحرف شده باشد تعداد جملاتی که باید نگه دارم بیشتر می‌شود

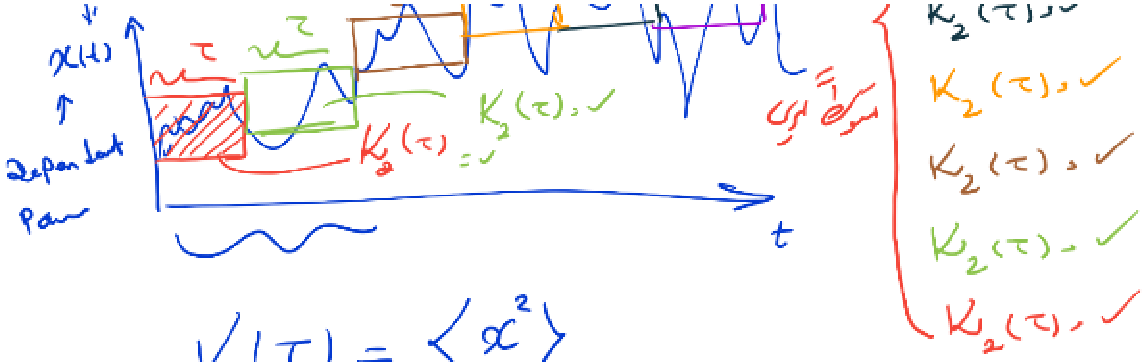
$K_1 = 0$

for a given τ

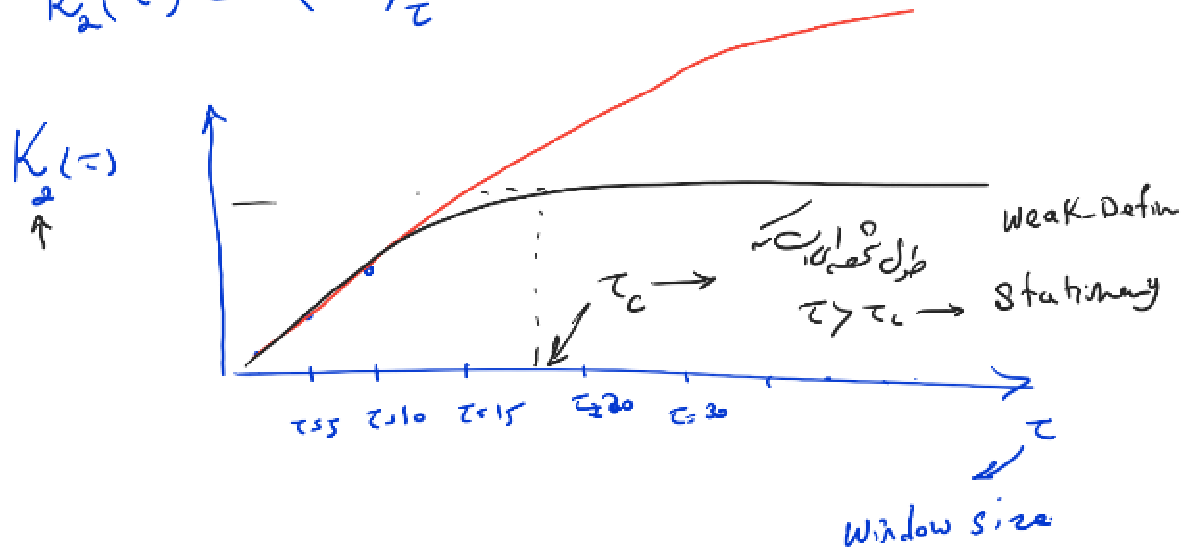
independent parameter $\quad K_2 = \langle x^2 \rangle$



$K_2(\tau) = \checkmark$



$$K_2(\tau) = \langle x^2 \rangle_\tau$$



$$\forall \; n \in R \qquad \underline{K_n(\tau)} = cts \quad \text{for} \quad \tau > \tau_c$$

$\tau_c \leftarrow$ ثابت زمانی سیستم است

Weak_Definition $\quad \underline{K_2(\tau > \tau_c) = cts}$

$$P(A_\mu) = \hat{\mathcal{L}} P_G(A_\mu)$$

$$\langle f \rangle = \int d^N A \; p(\vec{A}) \, f(\vec{A})$$

$$\int d^N A \left[\hat{\mathcal{L}} P(A_\mu) \right] f(\vec{A})$$

$$= \int d A \left[\mathcal{L}_G \cdots \right]$$

Using - Integration by-Part

$$\langle f \rangle = \int d^N A \left[\hat{\mathcal{L}}^{+} f(\vec{A}) \right] P_G(A_\mu)$$

$$\langle f \rangle = \left\langle \hat{\mathcal{L}}^{+} f(\vec{A}) \right\rangle_G$$

$$\hat{\mathcal{L}}^{+} \equiv \exp\left[\sum_{n\geq 3}^{\infty} \frac{(+1)^n}{n!} \left\{ \sum_{\mu_1=1}^{N} \cdots \sum_{\mu_n=1}^{N} K^{(n)}_{\mu_1 \cdots \mu_n} \frac{\partial^n}{\partial A_{\mu_1} \partial A_{\mu_2} \cdots \partial A_{\mu_n}} \right\} \right]$$

for Gaussian Process $\hat{\mathcal{L}} = 1$ — $\hat{\mathcal{L}}^{+} = 1$

$$\langle f \rangle = \int d^N A \; P(A_\mu) \, f(A)$$

$$= \int d^N A \; P_G(A_\mu) \, f(A)$$

$$\langle f \rangle = \langle f(A) \rangle_G$$

← f. / درجلسه بعد چند مثال را می‌بینیم