جلسه ۱۷/۱/۰۰

بسم الله الرحمن الرحیم

مباحثی که بررسی خواهیم کرد عبارتند از

→ ** Correlation function ←

** Weighted and Un-weighted Correlation

Value → Correlation Coefficient عدد گزارش می‌دهد

Value → Correlation function تابع است

به ازای عمل استفاده از تابع همبستگی بیشتر از صرفا ضریب همبستگی است.

$$f(x) \longrightarrow \{x - - \} \longrightarrow \{f - - \}$$

Cross-Correlation

همبستگی جزئی

$$C_{xy}(x,t;y,t') \equiv \langle [x(t)-\langle x\rangle][y(t')-\langle y\rangle]\rangle_{ens}$$

$$\{x_1, x_2, - \; x(t) \quad x_N\}$$

$$\{y_1, y_2 \; . y(t) \quad y_N\}$$

Auto-Correlation

خود همبستگی → $$C_{xx}(x,t;\underline{x,t'}) = \langle [x(t)-\langle x\rangle][x(t')-\langle x\rangle]\rangle_{ens}$$

تحلیل ثابت

Ex 1: for a stationary / Isotropic / Homogeneous

ای → T(t) زمان

همسانگرد → T(θ) زاویه

همگن → T(r) مکان

$$T(t), T(\theta), T(r)$$

$\langle T \rangle , \sigma_T \leftarrow \underset{G}{p(T)} \leftarrow$ برای متغیر راندوم

$\sigma_T(t) = cts$

$\sigma_T(\theta) = cts$

$\sigma_T(r) = cts$

فرآیند

مانا

می‌گرد

ممکن

Enemble average → Time average

Angular avera

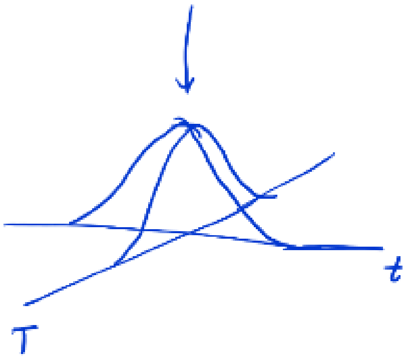
location averag.

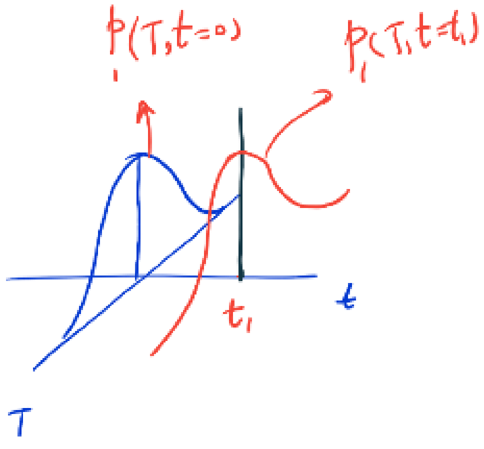
از آنجایی که اغلب فرآیندها خصوصاً در طبیعت (دنیای واقعی) وابسته به زمان در نظر می‌گیریم

"Time" "Stationary" = Angle (Direction) Isotropy = location Homogeneity

سری زمانی — Time-Series

$p_1(T, t) \rightarrow p(T)$





فرآیند ماناست → $p_1(T, t) = p_1(T)$ one-Point

تعریف ضعیف Weak Definition

N-Joint PDF

p (T 1 ; T 1 T t ; T t)

$$\to P_N(T_1, t_1; T_2, t_2; T_3, t_3; \dots; T_N, t_N)$$

$$= P_N(T_1, t_1+\tau; T_2, t_2+\tau; T_3, t_3+\tau, \dots T_N, t_N+\tau)$$

ما عملاً نمی‌توانیم این کمیت را تعیین کنیم.

Strong Definition for Stationarity

"$\underbrace{\text{Two-Point Statistics}}$ and Stationarity"

↓

Correlation

$$\{x_1, t_1; x_2, t_2, x_3, t_3 \dots \quad x_N, t_N\}$$

$$C(x_i, t_i; x_j, t_j) = \langle [x_i - \langle x_i\rangle_{en}][x_j - \langle x_j\rangle_{en}]\rangle_{ens}$$

$$\boxed{\text{If} \to \langle x_i\rangle = \langle x_j\rangle = 0}$$

به کلیت مسئله خیلی کم نمی‌کند.

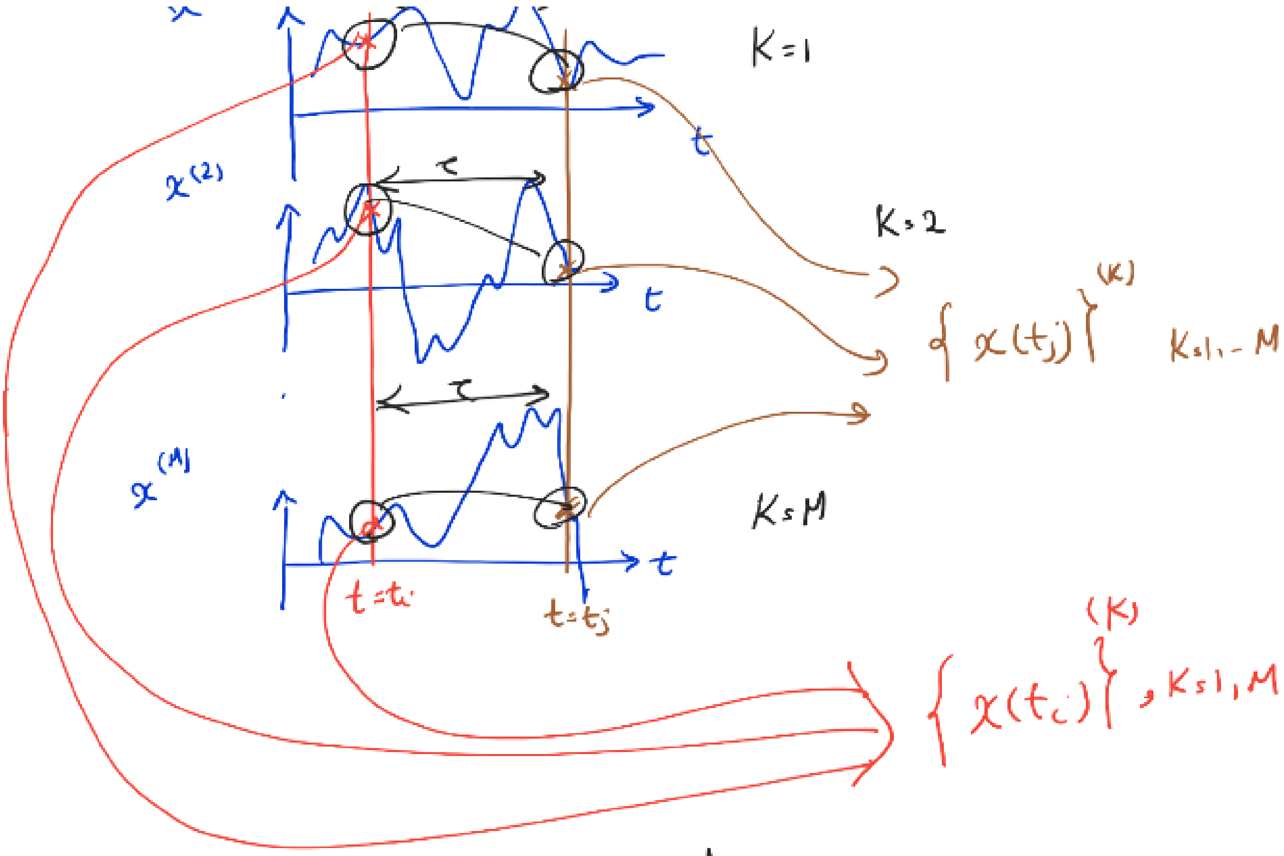
$$C \equiv \langle x_i x_j\rangle_{ens} = \langle x(t_i)\, x(t_j)\rangle_{ens}$$

مانا
تعریف ضعیف

$$\to C = \langle x(t_i)\, x(t_j)\rangle_{ens} = \langle x(t_i+\Delta)\, x(t_j+\Delta)\rangle_{ens}$$

$$C(t_i, t_j) = \langle x(t_i)\, x(t_j)\rangle_{ens}$$

$$\xleftrightarrow{\tau}$$



$$C(x(t_i), x(t_j)) = \frac{1}{M}\sum_{K=1}^{M} x^{(K)}(t_i)\, x^{(K)}(t_j)$$

↓ Stationarity

$$C(x(t_i), x(t_j)) \overset{\text{Sta.}}{=} C(x(\underline{t_i+\Delta})\, x(\underline{t_j+\Delta}))$$

$$\underline{\underline{C_x(t_i, t_j)}} \overset{\text{Sta.}}{=} C_x(|\overbrace{t_i - t_j}^{\equiv \tau}|) = \underline{\underline{C_x(\tau)}}$$

function

$$|t_i - t_j| = \tau$$

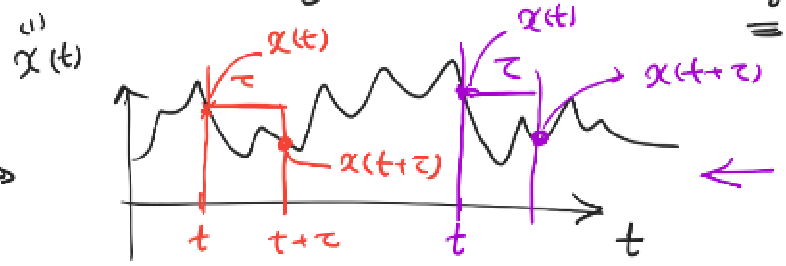
$$\Updownarrow \quad |t - (t+\tau)| = \tau$$

$$C_{xx}(\tau) = \langle \underbrace{x(\overset{t_i}{t})}_{x_i}\, \underbrace{x(\overset{t_j}{t+\tau})}_{x_j} \rangle_{\underline{\underline{t}}} \quad t \leftrightarrow \text{ens}$$

Auto-Correlation function (stationary)

Cross-Correlation function (Stationary)

$$C_{xy}(\tau) = \langle x(t)\, y(t+\tau) \rangle_t$$



$$C_{xx}(\tau) = \frac{1}{\underset{\rightarrow}{N-\tau}} \sum_{l=1}^{\boxed{N-\tau}} x(l)\, x(l+\tau)$$

for $\tau = 3$ → $C_x(\tau=3) = \langle x(t)\, x(t+\tau) \rangle_t$

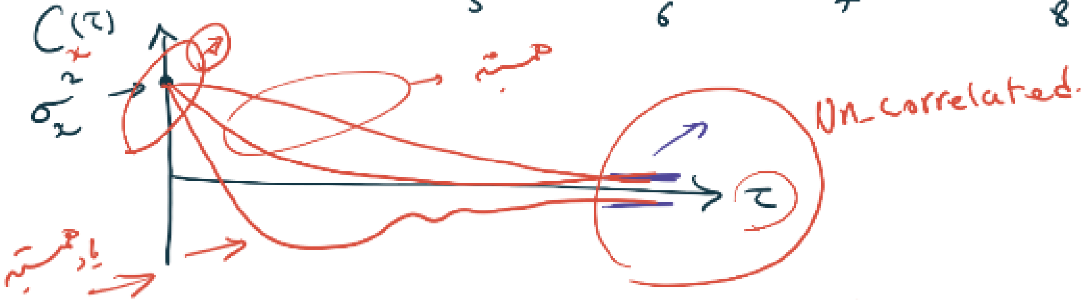
$$= \langle x(t)\, x(t+3) \rangle_t$$

$$C_x(3) = \frac{1}{\underset{8}{\boxed{11-3}}} \Big[\underbrace{x_1\, x_4}_{1} + \underbrace{x_2\, x_5}_{2} + \underbrace{x_3\, x_6}_{3} + \underbrace{x_4\, x_7}_{4}$$

$$+ \underbrace{x_5\, x_8}_{5} + \underbrace{x_6\, x_9}_{6} + \underbrace{x_7\, x_{10}}_{7} + \underbrace{x_8\, x_{11}}_{8} + x_9\, \cancel{x_{12}} \Big]$$

N=11

Auto-correlation



① $\lim C_x(\tau) = 0 = \langle x(t)\, x(t+\infty) \rangle =$ خیلی ضعیف

هیچ همبستگی وجود ندارد

Un_Correlated

② $C_x(\tau=0) = \langle x(t)\,x(t)\rangle = \sigma_x^2$

③ Intermediate Regime

وقتی $\langle x\rangle = 0$

$$C_x(\tau) = \langle x(t)\,x(t+\tau)\rangle_t = \langle\, [\,\underset{\downarrow\, t}{x(t)-\langle x\rangle}\,]\,[\,x(t+\tau)-\underset{t+\tau}{\langle x\rangle}\,]\,\rangle$$

$$= \int dx(t)\,dx(t+\tau)\;\; x(t)\,x(t+\tau)\;p_2(x(t);x(t+\tau))$$

Ex2: $C(t_i,t_j) \overset{?}{=} C(|t_i,t_j|)$

$$\downarrow$$

$$C(t_i,t_j) = \begin{cases} C(t_i,t_j) & \text{for } |t_i-t_j| < \tau' \\ C(\tau) & \text{for } |t_i-t_j| > \tau' \end{cases}$$

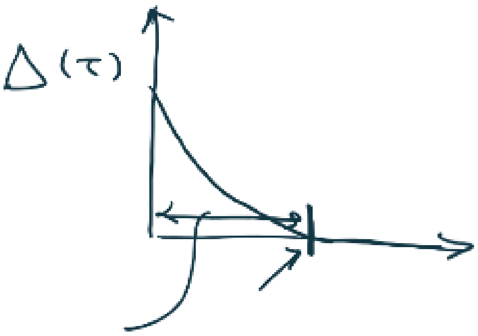
$\longrightarrow$

بعد از گذشت مدتی $\tau \gg \tau'$ ← سیستم مانا می‌شود

$$0 \neq \Delta(\tau) \equiv \int dx(t)\,dx(t+\tau)\,\Big|\,\underbrace{p_2(x(t);x(t+\tau)) - p_1(x(t))\,p_1(x(t+\tau}_{\neq}\Big|$$

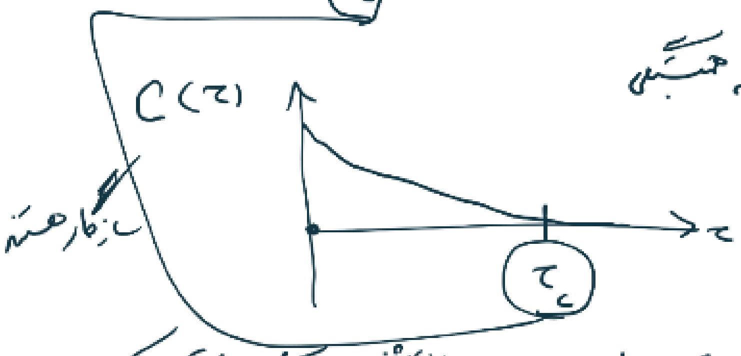
if → Un-correlated →

$$p_2(x(t); x(t+\tau)) = \underline{p_1(x(t))}\,\underline{p_1(x(t+\tau))}$$



$\boxed{\tau_c}$ ≡ characteristic time scale.

زمان مشخصه همبستگی



با کمک تابع همبستگی و یا تابع اتلاف هسته می‌توان طول (زمان) مشخصه همبستگی را تعیین کرد.

Classification of Stochastic Processes

طبقه‌بندی فرآیندهای تصادفی

نوع همبستگی را نوع تابع توزیع آماری پیروی کند

① Dependent Processes فرآیندهای وابسته

② Completely Random processes فرآیندهای کاملا کاتوره‌ای

(3) Markov Processes — فرایندهای مارکوف

$\{x_1, x_2, \ldots \quad x_N\}$

$t_1 < t_2 < t_3 \ldots < t_N$

فرآیند وابسته

N-Joint pdf

$$p_N(\underbrace{x_N, t_N}; \overbrace{x_{N-1}, t_{N-1}}; \underbrace{x_{N-2}, t_{N-2}}; \ldots ; x_1, t_1)$$

$$= p(\underline{x_N, t_N} \mid \underline{x_{N-1}, t_{N-1}}; \underline{x_{N-2}}, t_{N-2}; \ldots ; \underline{x_1, t_1})$$

$$\times p(\underline{x_{N-1}, t_{N-1}} \mid \underline{x_{N-2}}, \underline{t_{N-2}}; \ldots ; \underline{x_1}, t_1)$$

$\times$

$\vdots$

$$p(\underline{x_2, t_2} \mid \underline{x_1, t_1}) \times p(\underline{x_1, t_1})$$

حافظه ندارد

$$N \begin{cases} p_N(x_N, t_N; x_{N-1}, t_{N-1}; \ldots ; x_1, t_1) = \\ p(x_N, t_N)\, p(x_{N-1}, t_{N-1}) \times \ldots \times p(x_1, t_1) \end{cases}$$

$N=2 \rightarrow \quad p_2(x_2, t_2; x_1, t_1) = p(x_2, \underline{t_2})\, p(x_1, \underline{t_1})$

$$P_N(x_N,t_N;x_{N-1},t_{N-1};\ldots;x_1,t_1)$$

$$=\underline{p(x_N,t_N|x_{N-1},t_{N-1})}\times p(x_{N-1},t_{N-1}|x_{N-2},t_{N-2})\ldots$$

$$\times\, p(x_2,t_2|x_1,t_1)\;p(x_1,t_1)$$

N-Joint conditional PDF

$$\boxed{\begin{aligned}&p(x_N,t_N|\underbrace{x_{N-1},t_{N-1}}_{3};\underbrace{x_{N-2},t_{N-2};\ldots;x_1,t_1}_{3})\\&=\underline{p(x_N,t_N|x_{N-1},t_{N-2})}\end{aligned}}$$

→ شرط لازم

→ Chapmann–Kolomugrov Condition

$\boxed{N=3}$

$$p_3(x_3,t_3;x_2,t_2;x_1,t_1)$$

حاشیه‌ای

$$p_2(x_3,t_3;x_1,t_1)=\int dx_2\;p_3(x_3,t_3;x_2,t_2;x_1,t_1)$$

$$=\int dx_2\;\underline{p_2(x_3,t_3|x_2,t_2;x_1,t_1)}\,p_1(x_2,t_2|x_1,t_1)$$

$$\times\,p_1(x_1,t_1)$$

چه شرطی برقرار است

$$p(x_3,t_3;x_1,t_1) \overset{?}{=} \int dx_2\, \overbrace{p_1(x_3,t_3|x_2,t_2)}\, p_1(x_2,t_2|x_1,t_1)\, \underline{p_1(x_1,t_1)}$$

$$\frac{p_2(x_3,t_3;x_1,t_1)}{p_1(x_1,t_1)} \overset{?}{=} \int dx_2\, p_1(x_3,t_3|x_2,t_2)\, p_1(x_2,t_2|x_1,t_1)$$

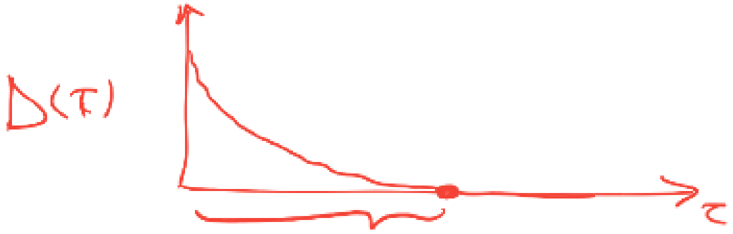
$$\boxed{p_1(x_3,t_3|x_1,t_1) \overset{?}{=} \int dx_2\, p_1(x_3,t_3|x_2,t_2)\, p_1(x_2,t_2|x_1,t_1)}$$

$$t_3-t_2=t_2-t_1=\tau$$

محک عملی

$$p(x_3,t+2\tau|x_1,t) \overset{?}{=} \int dx_2\, p_1(x_3,t+2\tau|x_2,t+\tau)\, p_1(x_2,t+\tau|x_1,t)$$

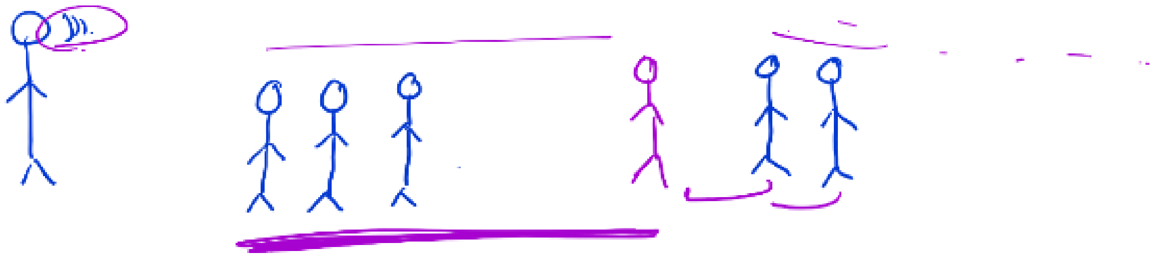
$$\Delta(\tau) \equiv \int dx_1\, dx_3 \left| p(x_3,t+2\tau|x_1,t) - \left[\int dx_2\, p_1(x_3,t+2\tau|x_2,t+\tau)\, p_1(x_2,t+\tau|x_1,t)\right]\right|$$



$\tau = \tau_{Markov}$

تعیین طول شبکه

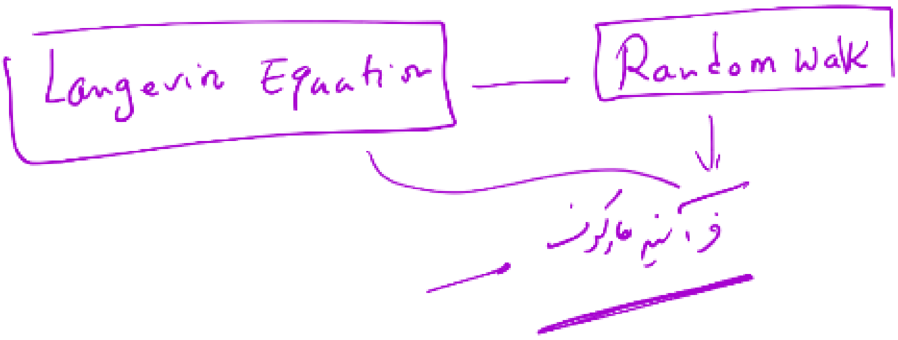
بیشترین طول زمانی، مقیاس زمانی‌ای که اطلاعات بصورت مستقیم منتشر می‌شود



τ_{Markov}

$\downarrow$

$\tau_{Markov} \xrightarrow[??]{??} \tau_{Causal}$



Langevin Equation — Random walk

فرآیند مارکوف

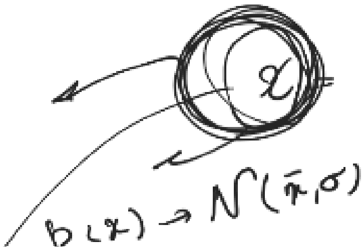
اریب

Weighted

Un-Weighted

در جلسه آینده تمرکز م

Centeral Limit Theorem



$b(x) \to \mathcal{N}(\bar{x},\sigma)$

$$\frac{1}{N}\sum_{i=1}^{N} y_i = \frac{1}{N}\left[\underbrace{\overset{\downarrow}{y_1} + \overset{\downarrow}{y_2} \cdot \quad \overset{\downarrow}{y_N}}\right]$$

$$(X^T \cdot C^{-1} \cdot X)$$

$$\boxed{p(x_1, x_2, \ldots)} \longrightarrow \boxed{p_2(x_1, x_2) = e^{\frac{\quad}{2}}}$$

$M=2$

$$\mathrm{Cov} = C = \begin{bmatrix} \langle x_1 x_1 \rangle & \boxed{\langle x_1 x_2 \rangle} \\ \boxed{\langle x_2 x_1 \rangle} & \langle x_2 x_2 \rangle \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_{x_1}^2 & \\ & \sigma_{x_2}^2 \end{bmatrix}$$

فرض کنید $\langle x_2 x_1 \rangle = \langle x_1 x_2 \rangle = 0$

$$p_2(x_1, x_2) = e^{\frac{-(x_1 \; x_2) \cdot \begin{bmatrix} \sigma_{x_1}^2 & \\ & \sigma_{x_2}^2 \end{bmatrix}^{-1} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}}{2}}$$

$$C = \begin{bmatrix} \sigma_{x_1}^2 & 0 \\ 0 & \sigma_{x_2}^2 \end{bmatrix} \longrightarrow C^{-1} = \frac{1}{\sigma_{x_1}^2 \sigma_{x_2}^2} \begin{bmatrix} \sigma_{x_2}^2 & 0 \\ 0 & \sigma_{x_1} \end{bmatrix}$$

$$\boxed{C^{-1} = \begin{bmatrix} \frac{1}{\sigma_{x_1}^2} & \\ & \frac{1}{\sigma_{x_2}^2} \end{bmatrix}}$$

$$p_2(x_1, x_2) = \exp\left[-\frac{1}{2} (x_1 \; x_2) \begin{bmatrix} \frac{1}{\sigma_{x_1}^2} & 0 \\ 0 & \frac{1}{\sigma_{x_2}^2} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right]$$

$$= \exp\left[-\frac{1}{2} (x_1 \; x_2) \begin{pmatrix} \frac{x_1}{\sigma_{x_1}^2} \\ \frac{x_2}{\sigma_{x_2}^2} \end{pmatrix} \right]$$

$$= \exp\left[-\tfrac{1}{2}\left(\frac{x_1^2}{\sigma_{x_1}^2} + \frac{x_2^2}{\sigma_{x_2}^2}\right)\right]$$

$$p(x_1, x_2) = e^{-\frac{x_1^2}{2\sigma_{x_1}^2}} \, e^{-\frac{x_2^2}{2\sigma_{x_2}^2}}$$

$$= p(x_1) \times p(x_2)$$

$$\boxed{p(x_1) = e^{-\frac{x_1^2}{2\sigma_{x_1}^2}}}$$

$$\boxed{p(x_2) = e^{-\frac{x_2^2}{2\sigma_{x_2}^2}}}$$

$$\boxed{p_a(x_1, x_2) = p(x_1)\, p(x_2)}$$

همبستگی بین x_1 و x_2 وجود ندارد

هیچ تأثیری روی شکل $p(x_1)$, $p(x_2)$ ندارم

$\rho = 0$

شکل

صحبت کردیم