In the name of God

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ADVANCED STATISTICAL MECHANICS I

Exercise Set 8

(Due Date: 1402/10/16)

1. Heat capacity: For phase transition, the value of C_V versus temperature (as external parameter) diverges. As an illustration, for Bose-Einstein condensation, at $T = T_c$, we obtain a divergency for C_V. Explain the physical concept of the mentioned behavior.

2. The Probability of having n particles in kth state with energy ϵ_k is given by $P_k(n)$. This probability can be written as occupation number, $\langle n_k \rangle$ for Maxwell-Boltzmann, Bose-Einstein (BE) and Fermi-Dirac (FD) statistics. According to the explicit form of $P_k(n)$, deduce that the BE statistics possesses the Bose enhancement, while the FD has Pauli blocking.

3. Determine the following thermodynamical potentials of Ultra-relativistic Bose Gas: F, H, G. Also Compute C_V and C_P for mentioned system.

4. For an ideal Bose gas, we obtained that at $T = T_c$, the fugacity is equal to one ($z = 1$), accordingly, we can determine the value of T_c (see the lecture note and notice to $N = \int_0^\infty d\epsilon g(\epsilon) n_{BE}(\epsilon)$, where $n_{BE}(\epsilon)$ is called BE distribution given by $n_{BE}(\epsilon) = \frac{1}{\exp(\beta(\epsilon-\mu))-1}$, for $T = T_c$ we have

$$N = \int_0^\infty d\epsilon g(\epsilon) \frac{1}{\exp(\frac{\epsilon}{k_B T_c} - 1)}$$

Now consider, our Bose system contains two level of energy, the particles in ground state have $\epsilon_0 = p^2/(2m)$ and those particles in excited state have $\epsilon = \epsilon_0 + \Delta$, for this case, compute T_c (Hint: at first determine the density of sate for the ground state $g_0(\epsilon)$ and for excited state $g_{\text{excited}}(\epsilon)$, then set $z = 1$).

5. According to the statistical definition of pressure, determine the equation of state parameter of ideal photon gas.

Good luck, Movahed