In the name of God

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ADVANCED STATISTICAL MECHANICS I

Exercise Set 5

(Due Date: 1402/09/01)

1. For speed distribution function Maxwell-Boltzmann, compute the **most probable speed**, **mean speed** and $\langle v^2 \rangle_{speed}$. Explain the physical meaning pf them and compare to each other.

2. Using micro-canonical ensemble, prove equipartition theorem (Hint: See chapter 7 Greiner's book).

3. Suppose that we have a system with 3-level energy. Based on Boltzmann statistics and canonical ensemble, compute the $\langle E \rangle$. What happens for $\beta \to 0$ and $\beta \to \infty$.

4. In canonical ensemble determine the $\sigma_T^2 \equiv \langle (T - \langle T \rangle)^2 \rangle$ and $\sigma_V^2 \equiv \langle (V - \langle V \rangle)^2 \rangle$. (Hint: see paper with arXiv:1507.05662)

5. Application of Virial theorem: For interacting system, we have $\mathcal{H} = \sum_{i=1}^{N} \frac{\vec{p}_i^2}{2m} + \sum_{i<i} \mathcal{U}(|\vec{q}_i - \vec{q}_j|)$, it turns out that the equation of state is different from Ideal gas. According to the Virial theorem we have

$$PV = Nk_BT + \frac{1}{3} \left\langle \sum_{i=1}^{N} \vec{r}_i . \frac{\partial \mathcal{U}}{\partial \vec{r}_i} \right\rangle$$

and we can define $\mathcal{U}(r) \equiv \mathcal{U}(|\vec{q}_i - \vec{q}_j|)$ and $r \equiv |\vec{q}_i - \vec{q}_j|$. Show that

$$PV = Nk_BT \left[1 - \frac{N(N-1)}{6VNk_BT} \int d^3r \; r \frac{\partial \mathcal{U}}{\partial r} g(r) \right]$$

here $g(r)$ is the probability of finding two particle separated by distance r (g of r).

6. Solve exercise of chapter 3 (R. K. Pathria-3th edition): Q5, Q8, Q15, Q20, Q27, Q29, Q35, Q36, Q40, Q42

Good luck, Movahed