In the name of God

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ADVANCED TOPICS IN STATISTICAL PHYSICS II

Exercise Set 6

(Date Due: 1394/03/01)

1. Generate Gaussian data set with given correlation function (e.g. the shape of correlation function follows a scale invariant behavior $(C \sim \tau^{-\gamma})$ and show that your generated data is Gaussian and has got mentioned correlation function.

2. Generate Non-Gaussian data set with desired correlation function. As an example suppose that the shape of correlation function follows a scale invariant behavior $(C \sim \tau^{-\gamma})$ and show that your generated data is Non-Gaussian and has got mentioned correlation function.

3. According to following definition:

$$\int_{-\infty}^{+\infty} x^n \exp[-(x-\beta)^2]dx = (2i)^{-n}\sqrt{\pi}H_n(i\beta)$$

where H_n are the Hermite polynomials show that:

$$M_n(x',t,\tau) = \left[-i\sqrt{D^{(2)}(x',t)\tau}\right]^n H_n\{\frac{1}{2}iD^{(1)}(x',t)\sqrt{\tau/D^{(2)}(x',t)}\}$$

also show that above equation causes to correct function for $D^{(n)}$.

4. Calculate the Moments: Using Green's function approach show:

   A :
   $$M_i(t) = \langle x_i(t) \rangle = G_{ij}x_j$$

   B :
   $$\sigma_{ij} = \langle [x_i(t) - \langle x_i \rangle][x_j(t) - \langle x_j \rangle] \rangle = \int_0^t G_{ik}(t')G_{js}(t')g_{ks}$$

   C :
   $$\dot{\sigma}_{ij} = -\xi_{ik}\sigma_{kj} - \xi_{jk}\sigma ki + g_{ij}$$

   D :
   $$\ddot{\sigma}_{ij} = -\xi_{il}G_{lk}G_{js}g_{ks} - G_{ik}\xi_{jl}G_{ls}g_{ls}$$

5. Using the value of $D^{(1)}, D^{(2)}, D^{(3)}, D^{(4)}$, compute $\langle x^4 \rangle$ as a function of $\langle x^3 \rangle$ and $\langle x^2 \rangle$ for data that you have.

6. By computing the $D^{(1)}$ and $D^{(2)}$ for $\Delta x \equiv x(t+\tau) - x(t)$, compute the correlation function, $C_x(\tau) = \langle x(t+\tau)x(t) \rangle$. Compare your results with that of given directly by data.

Good luck, Movahed