In the name of God

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ADVANCED TOPICS IN STATISTICAL PHYSICS II

Exercise Set 4

(Date Due: 1395/01/20)

1. Suppose that for an isotropic stochastic field in D-dimension $\alpha \equiv \frac{f}{\sigma_0}$, $\vec{\eta} = \vec{\nabla}\alpha$ and $\xi = \nabla^2\alpha$ Show that:

$$\langle \alpha^2 \rangle = 1$$

$$\langle \eta_1^2 \rangle = \langle \eta_2^2 \rangle = \langle \eta_3^2 \rangle = \frac{1}{D}\frac{\sigma_1^2}{\sigma_0^2}$$

$$\tilde{\xi}_{ij} \equiv \xi_{ij} + \frac{1}{D}\frac{\sigma_1^2}{\sigma_0^2}$$

$$\langle \tilde{\xi}_{11}^2 \rangle = \langle \tilde{\xi}_{22}^2 \rangle = \langle \tilde{\xi}_{33}^2 \rangle = \frac{3}{D(D+2)}\frac{\sigma_2^2}{\sigma_0^2}\left(1 - \frac{D+2}{3D}\gamma^2\right)$$

$$\langle \tilde{\xi}_{11}\tilde{\xi}_{22} \rangle = \langle \tilde{\xi}_{11}\tilde{\xi}_{33} \rangle = \langle \tilde{\xi}_{22}\tilde{\xi}_{33} \rangle = \frac{1}{D(D+2)}\frac{\sigma_2^2}{\sigma_0^2}\left(1 - \frac{D+2}{D}\gamma^2\right)$$

$$\langle \tilde{\xi}_{12}^2 \rangle = \langle \tilde{\xi}_{13}^2 \rangle = \langle \tilde{\xi}_{23}^2 \rangle = \frac{1}{D(D+2)}\frac{\sigma_2^2}{\sigma_0^2}$$

2. Compute $\langle |\eta_i| \rangle$ for 1 and 2 and 3 dimensions.

3. Show that using new variables, x, y, z as follows:

$$x \equiv \frac{\sigma_0}{\sigma_2}\left(\sum_i \xi_{ii} + \frac{\sigma_1^2}{\sigma_0^2}\alpha\right)$$

$$y \equiv -\frac{\sigma_0}{\sigma_2}\frac{\xi_{11} - \xi_{22}}{2}$$

$$z \equiv -\frac{\sigma_0}{\sigma_2}\frac{\xi_{11} + \xi_{22} - 2\xi_{33}}{2}$$

the covariance matrix, $C \equiv \langle A \bigotimes A \rangle$ is diagonal. Here $A : (\alpha, \eta_1, \eta_2, \eta_3, x, y, z, \xi_{12}, \xi_{13}, \xi_{23})$

4. Using the new variables $\alpha, \eta_1, \eta_2, \eta_3, x, y, z, \xi_{12}, \xi_{13}, \xi_{23}$, compute:

$$\mathcal{L} \equiv \int \frac{e^{-\frac{1}{2}A^T.C^{-1}.A}}{(2\pi)^5\sqrt{Det(C)}}dA$$

Good luck, Movahed