Lesson No. بسم الله الرحمن الرحيم

① Classical (Ideal) Gas From Statistical Mechanics (Continuation)

استمرار الغاز المثالي من خلال الميكانيكا الاحصائية

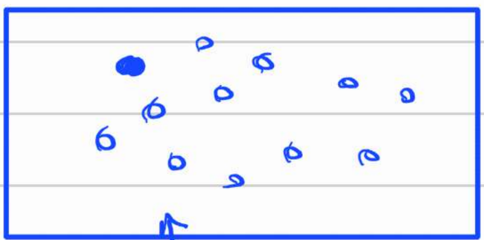
② The Concept of <u>Micro State</u> : Ω

المفهوم الفضاء الطور

① The Connetion point of thermodynamics and Statistical Mechan

is
$$S = K_B \ln \Omega$$

↑ The Number of Microsta



$N = \checkmark$

$V = \checkmark$

$P = \checkmark$

Without Intraction بدون تفاعل

$H = H_0$

$v \longrightarrow 0$ الحجم للجسيم

$V \neq 0$ حجم لصندوق

The accesible volume for each Particle

V يمكن وصل حجم لكل جسيم

$\boxed{\Omega \sim V^N}$, $S = K_B \ln \Omega$

$S = K_B \ln V^N$

$$\frac{P}{T} = \left(\frac{\partial S}{\partial V}\right)_{U,N} = \frac{\partial}{\partial V}\left(K_B \ln \Omega\right)$$

$$\frac{P}{T} = \frac{\partial}{\partial V}\left(K_B \ln V^N\right) = \frac{K_B N}{V}$$

$$\frac{P}{T} = \frac{K_B N}{V} \Rightarrow \boxed{PV = K_B N T}$$

What about other Thermodynamical properties ?

$\Omega = ?$ Cube : مكعب

3 Dimension

$$\boxed{V = L^3}$$

$$H = \sum_{i=1}^{N} \epsilon_i$$

ϵ_i : الطاقة الداخلية للجسم

$\epsilon_i = ?$ From <u>Quantum Mechanics</u>.

Planck const

$$\epsilon_i = \frac{h^2}{8mL^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

$n_x, n_y, n_z = 1, 2, \ldots$ لكل

mass of particle :

$$H = \sum_{i=1}^{N} \epsilon_i = \sum_{i=1}^{N} \frac{h^2}{8mL^2}\left(n_{x_i}^2 + n_{y_i}^2 + n_{z_i}^2\right)$$

$$H = \frac{h^2}{8mL^2} \sum_{i=1}^{3N} n_i^2$$

$n_i = 1, 2, \ldots$

$$\boxed{R \equiv \frac{H}{\left(h^2/8mL^2\right)} = \sum_{i=1}^{3N} n_i^2}$$

$R \equiv$ Total Number of energy cargo

which can be distributed to Particle

كل عدد الطاقة : تمكن توزيع الطاقة للجسيم

3N box

R = Balls

$$* \boxed{\Omega \equiv \frac{(R+(3N-1))!}{R!\,(3N-1)!}} *$$

Probability approach

$$R = \frac{\overset{cts}{H}}{\left(\frac{h^2}{8mL^2}\right)} \propto HL^2 = HV^{2/3}$$

$$\boxed{R \propto V^{2/3} H}$$

$V = L^3$ — $L = V^{1/3}$

$$R \propto \Omega \longrightarrow S = K_B \ln \Omega$$

$$S \propto K_B \ln R$$

$$S \propto K_B \ln HV^{2/3}$$

$$P = -\left(\frac{\partial H}{\partial V}\right)_{S,N}$$

$$\boxed{P = \frac{2}{3}\frac{H}{N}}$$

If $S = cts \Rightarrow H V^{2/3} = cts$

$$\boxed{P = \frac{2}{3}\frac{H}{V}}$$

For Ideal Gas

Energy Density.

$$H V^{2/3} = cts$$

$$P = \frac{2}{3}\frac{H}{N}$$

$$P V^{5/3} = cts$$

$$P V^{\gamma} = cts$$

$$\boxed{\gamma = \frac{C_p}{C_V}}$$

Adiabatic Process

$S = cts$

$$\boxed{\gamma = 5/3}$$

$\Omega = ?$

We found that

$$\Omega = \frac{(R + (3N-1))!}{R!\,(3N-1)!} \quad \text{and} \quad R \equiv \frac{H}{h^2/8mL^2} = \frac{U}{h^2/8mL^2}$$

$$\boxed{\ln N! = N \ln N - N}$$

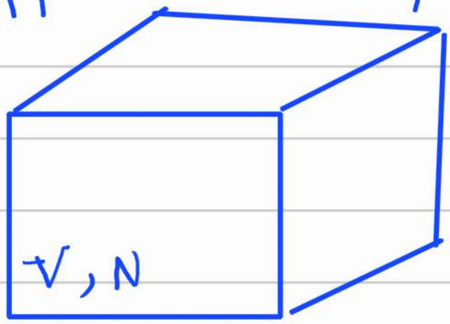
$$S = K_B \ln \Omega$$

For Ideal Gas

$$\boxed{S = NK_B \ln\left[\frac{V}{h^3}\left(\frac{4\pi m U}{3N}\right)^{3/2}\right] + \frac{3}{2} NK_B}$$

$$U = \frac{3}{2} NK_B T$$

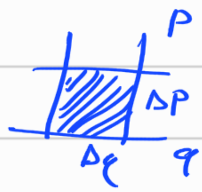
② Other approach To Compute Ω: Total Microstate



$\Delta p \Delta x \geq h$

$$\text{Phase space} \equiv \sum \equiv \int \frac{dq^{3N}\, dp^{3N}}{h^{3N}}$$

حجم الطور



q_1 The coordinate for First Particle :

q_2 〃 〃 〃 Second 〃

.

.

q_N — — — Nth Part

الزخم p momentum $p = mv$ سرعة

$p \to (P_x, P_y, P_z)$, $v \to (v_x, v_y, v_z)$

P_1 the momentum for First Particle

P_2 〃 〃 〃 Second 〃

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.

P_N 〃 〃 〃 Nth Particle

Example :

$V = L^3$

حجم للمكعب

$$\Sigma = \int \frac{d^{3N}q\, d^{3N}p}{h^{3N}} = \int_V d^{3N}q \int_{H \leq U} \frac{d^{3N}p}{h^{3N}} = \int_{\text{Shell}} \frac{d^{3N}q\, d^{3N}p}{h^{3N}}$$

قيد

Constraint limit

$H \leq U$

$$H = \sum H_i$$

$$H_i = \frac{P_i^2}{2m} = \frac{(m v_i)^2}{2m} = \frac{m^2 v_i^2}{2m} = \frac{1}{2} m v_i^2$$

$$P_i^2 = 2m H_i$$

(Energy , الطاقة

Shell $\equiv$ Considering the constraints For q and p

حجم

$$\Sigma = \frac{1}{h^{3N}} \int_V \overbrace{d^3q_1\, d^3q_2 \ldots d^3q_N}\ \int_{H \leq U} \underbrace{d^3p_1\, d^3p_2 \ldots d^3p_N}$$

$$= \frac{1}{h^{3N}} \left[\left(\int d^3q_1\right)\left(\int d^3q_2\right)\left(\int d^3q_3\right) \ldots \left(\int d^3q_N\right) \right] \times \ \parallel$$

$$= \frac{1}{h^{3N}} \left[\overbrace{V \times V \times \ldots V}^{N} \right] \times \int d^3p_1 \int d^3p_2 \ldots \int d^3p_N$$

$$\Sigma = \frac{1}{h^{3N}} V^N \times \int_{H \leq U} d^3p_1\, d^3p_2 \ldots d^3p_N$$

?