In the name of God

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MODERN PHYSICS

Exercise Set 1

~~(Due Date: 1402/12/10)~~ (Due Date: 1402/12/18)

1. According to the definition of thermal wavelength, $\lambda_T \equiv \left(\frac{h^2}{2\pi m k_B T}\right)^{1/2}$, compute the order of magnitude of:
A : The typical value of λ_T for mankind in the room temperature.
B : The typical value of λ_T for air molecules in the room temperature.
C : The typical value of λ_T for proton in the colliding experiment in CERN.
D : The typical value of λ_T for Hydrogen atom when the age of Universe was only 300,000 years old.
E : The typical value of λ_T for Hydrogen atom for interstellar medium .
F : The typical value of λ_T for Hydrogen atom when the age of Universe was only 300,000 years old.
G : Plot the λ_T for Hydrogen atom as a function of Temperature in the range of when the universe had only 300,000 years old till now. (Hint: $T(t = 300,000 years) = 2725$ K and temperature as the current era is about $T(t = 13.8 \times 10^{10} years) = 2.7255 \pm 0.0006$ K).

2. An straightforward mathematical modeling for wave-behavior of particle is given by a Gaussian function as follows:

$$|\psi(x)|^2 \sim e^{-\frac{x^2}{2\sigma^2}}$$

above function is associated with probability of finding a particle in position x. Plot above function for $\sigma = 0.001$, $\sigma = 0.01$, $\sigma = 0.1$, $\sigma = 1$, $\sigma = 10$ and $\sigma = 100$. (Hint: you can use Mathematica or Maple or Python to plot this function)

3. Lenard-Jones potential: In order to model the potential between molecules, a feasible function is so-called Lenard-Jones as:

$$\mathcal{U}(r) = \left[\frac{A}{r^{12}} - \frac{B}{r^6}\right]$$

Plot above function. Also investigate $\lim_{r\to 0}\mathcal{U}(r)$ and $\lim_{r\to\infty}\mathcal{U}(r)$. Suppose $A = B = 1$

4. For an Ideal Gas the Pressure and internal energy are given by:

$$PV = Nk_BT = nRT$$
$$U = \frac{3}{2}Nk_BT$$

but if we have non-Ideal gas for which the Hamiltonian is $\mathcal{H} = \mathcal{H}_0 + \mathcal{U}$, then above quantities are modified via:

$$P = nk_BT\left[1 - \frac{n}{2Dk_BT}\int d^Dr\, r\, \frac{d\,\mathcal{U}(r)}{dr}\, g(r)\right]$$
$$U = \frac{D}{2}Nk_BT + \frac{nN}{2}\int d^Dr\, r\, \mathcal{U}(r)\, g(r)$$

suppose that for $D = 3$ (3-dimension) and $g(r) = ((\sin(r)/r)^2 + 1)(1 - \exp(-r))$ and $\mathcal{U} = -\frac{1}{r}$, compare numerically the pressure and internal energy of interacting system with ideal gas and plot them as a function of Temperature.

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5. A simple molecular dynamic simulation:
Suppose that we have a box in 2-dimension with size $L = 1$ including 10 atoms whose radius equates to $d = 0.01$. Also suppose that the Hamiltonian is $\mathcal{H} = \sum_{i=1}^{10} \frac{\vec{p}_i^2}{2m}$. The collision between pairs and the walls are completely elastic. Simulate the evolution of them and make a movie from their evolution. (Hint: the initial conditions come from the Maxwell-Boltzmann distribution for velocity and the location of atoms are randomly selected inside the dox, also consider $T = k_B = m = 1$).

6. Equipartition theorem: Suppose that for a system in D-dimension with N particles, the Hamiltonian is given by:

$$\mathcal{H} = \sum_{i=1}^{DN} \left(\frac{p_i^{\xi}}{2m} + m q_i^{\gamma} \right)$$

Using equipartition theorem, show that:

$$E \equiv \langle \mathcal{H} \rangle = \frac{DN}{\xi} k_B T + \frac{DN}{\gamma} k_B T$$

7. All questions of chapter 1 for Krane, Kenneth S. Modern physics. John Wiley & Sons, 2019 must be answered.

8. Questions no. 3, 5, 6, 10,13 and 16 of chapter 1 for Krane, Kenneth S. Modern physics. John Wiley & Sons, 2019.

Good luck, Movahed