باسمه تعالی

جمع بندی ماتریس فیشر.

(1) $$\mathcal{L}_{Relative} = e^{-\frac{\Delta\chi^2}{2}} = e^{-\frac{\Delta^T \cdot C^{-1} \cdot \Delta}{2}}$$

$\Delta \equiv y - y_{theo}$, $C \equiv$ Covariance of observed Data

(2) $$\mathcal{L}_{Rela} = \sqrt{\frac{Det\, F}{(2\pi)^M}}\, e^{-\frac{\Delta\theta^T \cdot F \cdot \Delta\theta}{2}}$$

$$\mathcal{L}_{Relative} = e^{-\frac{\Delta\chi^2}{2}} = \sqrt{\frac{Det(F)}{(2\pi)^M}}\, e^{-\frac{\Delta\theta^T \cdot F \cdot \Delta\theta}{2}}$$

$$\ln \mathcal{L}_{Rel} = -\frac{\Delta\chi^2}{2} = -\frac{\Delta\theta^T \cdot F^{-1} \cdot \Delta\theta}{2} + \ln(\quad)$$

(3) $$\boxed{F_{ij} = -\left\langle \frac{\partial^2 \ln \mathcal{L}_{Rel}}{\partial\theta_i \partial\theta_j} \right\rangle = \frac{1}{2}\left\langle \frac{\partial^2 \Delta\chi^2}{\partial\theta_i \partial\theta_j} \right\rangle}$$

حال برای کل سمت راست خواهیم داشت:

$$④\ f \equiv -\ln \mathcal{L}_{Rel} = +\frac{1}{2}\left(\Delta^T \cdot C^{-1} \cdot \Delta\right)$$

چون عدد است

$$f = \frac{1}{2}\,\mathrm{Tr}\left(\Delta^T \cdot C^{-1} \cdot \Delta\right)$$

$$\frac{\partial f}{\partial \theta_i} = \mathrm{Tr}\left(C^{-1} \frac{\partial(\Delta^T \Delta)}{\partial \theta_i}\right)$$

$$\left\langle \frac{\partial f}{\partial \theta_i} \right\rangle = \mathrm{Tr}\left(C^{-1} \left\langle (\Delta^T \cancel{\Delta})_{,i} \right\rangle \right)$$

صفر می‌شود چرا

$$\left\langle Y_{,i} (y_i - Y_i) \right\rangle = Y_{,i} \left\langle (y_i - Y_i) \right\rangle = 0$$

$$\frac{\partial^2 f}{\partial \theta_i \partial \theta_j} = \mathrm{Tr}\left[C^{-1} (\Delta^T \Delta)_{,ij} \right]$$

$$= \mathrm{Tr}\left[\Delta^T_{,i} \cdot C^{-1} \cdot \Delta_{,j} \right]$$

$$\boxed{F_{ij} = \mathrm{Tr}\left[\left\langle \Delta^T_{,i} \cdot C^{-1} \cdot \Delta_{,j} \right\rangle \right]}$$

$$\langle F_{ij} \rangle = Tr \left[\left\langle \frac{\partial Y_{Th}}{\partial \theta_i}^T \cdot C^{-1} \cdot \frac{\partial Y_{Th}}{\partial \theta_j} \right\rangle \right]$$

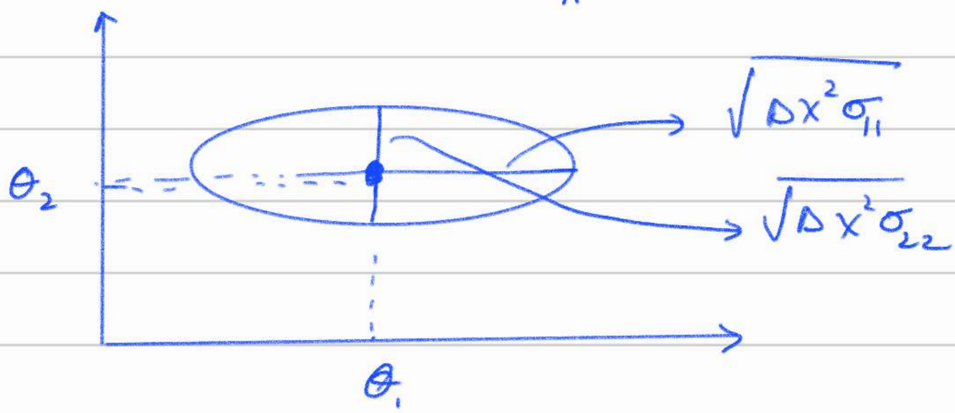
(5) $$\sigma_{\theta_i} = (F^{-1})_{ii}$$

(6) For $M=2$ $$\delta\theta^T . F . \delta\theta = \Delta\chi^2$$

$$\frac{\delta\theta_1^2}{\sigma_{11}^2(1-\rho^2)} + \frac{\delta\theta_2^2}{\sigma_{22}(1-\rho^2)} - \frac{2\rho}{(1-\rho^2)} \frac{\delta\theta_1}{\sigma_{11}} \frac{\delta\theta_2}{\sigma_{22}} = \Delta\chi^2$$

که بیضی دوران یافته است

(7) In Diagonal Case. $$\frac{\delta\theta_1^2}{\sigma_{11}^2} + \frac{\delta\theta_2^2}{\sigma_{22}^2} = \Delta\chi^2$$



(8) In General Case.

$$\delta\tilde{\theta} = P\delta\theta \quad , \quad P = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ +\sin\alpha & \cos\alpha \end{bmatrix}$$

$$\tan 2\alpha = \frac{2\rho\,\sigma_{11}\,\sigma_{22}}{\sigma_{22}^2 - \sigma_{11}^2}$$

$$\tilde{\sigma}_{11}^2 = \frac{\sigma_{11}^2 + \sigma_{22}^2}{2} + \left[\frac{(\sigma_{11}^2 - \sigma_{22}^2)^2}{4} + (\rho\sigma_{11}\sigma_{22})^2 \right]^{1/2}$$

$$\tilde{\sigma}_{22}^2 = \frac{\sigma_{11}^2 + \sigma_{22}^2}{2} - \left[\frac{(\sigma_{11}^2 - \sigma_{22}^2)^2}{4} + (\rho\sigma_{11}\sigma_{22})^2 \right]^{1/2}$$

(9) for a given Probability $P\%$

and $n =$ Dimension of Contour

$$P\% = \int_0^{\Delta\chi^2} d\Delta\chi^2 \, P_n(\Delta\chi^2) \longrightarrow \Delta\chi^2 = \checkmark$$

$$P_n(\Delta\chi^2) = \frac{1}{2^{n/2}\,\Gamma(n/2)} (\Delta\chi^2)^{\frac{n}{2}-1} e^{-\frac{\Delta\chi^2}{2}}$$

(10) Goodness of fit.

$$P_{<P_{obs}} = 2P_{\chi^2 > \chi^2_{obs}} = 2\int_{\chi^2 > \chi_{ob}}^{+\infty} d\chi^2 \, P(\chi^2; \nu)$$

$$\nu = N - M$$

for $\nu \gg 1 \rightarrow P(\chi^2; \nu) \rightarrow$ Gaussian $\sim e^{-\frac{(\chi^2 - \nu)^2}{2(2\nu)}}$

$$\left\{\begin{array}{l} \chi^2 = \nu \pm \sqrt{2\nu} \\ \chi^2_\nu = \dfrac{\chi^2}{\nu} = 1 \pm \sqrt{\dfrac{2}{\nu}} \end{array}\right\}$$