

In the name of God

# Department of Physics Shahid Beheshti University

## NUMERICAL ANALYSIS COURSE

### Exercise Set 4

(Due Date: 1403/08/17)

1. Joint PDF: For the input data set, compute

$$\Delta(\tau) \equiv \int dx_1 dx_3 |p(x_3, t + 2\tau; x_1, t) - \int dx_2 p(x_3, t + 2\tau | x_2, t + \tau) p(x_2, t + \tau | x_1, t) p(x_1, t)|$$

as a function of  $\tau$ . Explain your results.

2. According to Box-Muller algorithm, generate Gaussian random field with  $\sigma_0^2 = 2$  and  $\langle x \rangle = 3$ . Check your results by fitting a Gaussian function on the computed PDF of your generated data.
3. According to Von-Neumann method, generate a set of random data set in the range  $x \in [1 - 5]$  with PDF as:  $p(x) = \sin(x^2/100) + \frac{1}{\cos(x^3/100)} + x^{-3}$ .
4. PDF transformation: Suppose that in a black box a harmonic oscillator is oscillating and you made a series of snapshots randomly through time from the position of harmonic oscillator around the equilibrium. Determine the PDF of the location of the oscillator in the stationary case.
5. Suppose that  $x$  has the Pareto distribution,  $p(x) = \frac{a}{x^{a+1}}$  for  $1 \leq x < \infty$ . Find the probability density function of each of the following random variables:

**A** :  $y = x^2$ .

**B** :  $z = \frac{1}{x}$ .

**C** :  $T = \ln(x)$ .

Good luck, Movahed

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