Date: 1401/09/10

In the name of God

Department of Physics, Shahid Beheshti University

CRITICAL PHENOMENA

Midterm exam

(Time allowed: 3:00 hours)

1. Phase transition in 1d and 2d Ising models. According to energy cost to create a typical boundary between up and down spins, show that 1d Ising model has no non-trivial fixed point (i.e. $T_c = 0$), while for 2d Ising model there is a $T_c \neq 0$. (10 points)

2. Transfer matrix formalism: imagine that a periodic 1d Ising model whose coupling constants are J and K repeated successively in the string according to following figure

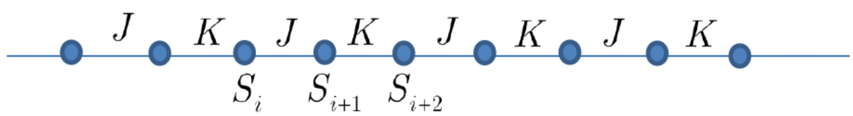

Figure 1: This figure shows what I want for question.

A: Write the Hamiltonian for this system. (5 points)
B: Determine the corresponding Transfer matrix. (10 points)

3. Mean field theory: For the XY mode, determine the scaling exponents for $M \sim t^{\beta}$ and $\chi \sim t^{\gamma}$. Here $t \equiv (T - T_c)/T$. (Hint: $\int_{-\pi}^{+\pi} d\theta e^{a(\cos(\theta))} = (2\pi)I_0(a)$)(20 points)

4. Mean field theory: For the following Hamiltonian:

$$\mathcal{H} = -\sum_{ij} J_{ij} S_i S_j - \epsilon \sum_{lmn} K_{lmn} S_l S_m S_n - H \sum_{q} S_q$$

where $\epsilon \ll 1$
A: Write the mean field correction to H. (10 points)
B: Compute the Critical temperature. (5 points)
C: Deduce the probable correction to a typical corresponding critical exponent compare to common Ising model. (5 points)

5. Ginzburg criterion: According to the Ginzburg criterion, one can evaluate the goodness of mean field theory, to this end suppose the Ising model with

$$\mathcal{H} = -J \sum_{ij} S_i S_j$$

and in the mean field theory, we can rewrite the above hamiltonian as $\mathcal{H} = \mathcal{H}_{\text{mean-field}} + \mathcal{H}_{\text{correction}}$, compute

$$\frac{\mathcal{H}_{\text{correction}}}{\mathcal{H}_{\text{mean-field}}}$$

by means of dimensional analysis. Deduce which dimension is good for? (Hint: Near critical point and for $t \to 0^+$, at first compute $G(r)$ or its Fourier transform, $\tilde{G}(k)$.) (15 points)

Good luck, Movahed