

SN	final exam (160)	Ex1 20	Ex2 12	Ex3 12	Ex4 5	Ex5 5	Ex6 12	Ex7 12	Ex8 12	Ex9 10	Ex10 12	Ex11 5	Ex12 10	Ex13 10	Ex14 5	Ex15 5	Ex16 5	Ex17 5	Ex18 5	Ex19 5	Ex20 5	Ex21 20	Ex22 5	Ex23 5	Ex24 5	Ex25 5	Ex26 5	Ex27 5	Ex28 10	Ex29 20	Ex30 10	Ex31 5	Ex32 15	project 2	
91516001	133	20	12	0	5	5	10	6	6	10	10	0	0	0	5	5	5	0	5	5	5	0	5	5	5	5	5	5	10	0	0	5	0	0	
91416027	123	20	12	6	5	5	12	10	0	10	12	5	0	0	5	5	5	0	5	5	5	15	5	5	5	5	5	5	0	10	10	5	0	2	
91416028	75	20	12	12	5	5	12	6	9	10	12	5	9	0	0	5	5	0	0	5	5	15	5	5	5	5	5	5	10	5	0	5	0	0	
91416046	121	20	12	0	5	5	12	6	9	10	12	5	5	4	5	5	5	0	5	5	5	15	5	5	5	5	5	5	0	10	10	5	0	2	
91515008	60	20	12	12	5	5	0	6	9	10	6	3	5	0	5	0	0	0	0	0	0	5	0	0	0	0	0	0	0	0	0	0	0	0	0
91416074	73	20	12	0	5	5	4	0	9	10	6	5	3	5	5	5	5	0	5	5	5	10	5	5	5	5	5	5	5	5	0	5	0	0	
91416068	112	20	12	0	5	5	12	10	10	10	12	5	5	5	5	5	5	0	5	5	5	10	5	5	5	5	5	5	0	8	10	0	0	0	
91516012	148	20	12	12	5	5	12	12	10	10	10	5	10	10	5	5	5	5	5	5	5	12	0	0	5	5	5	5	10	10	0	0	15	2	
91516034	115	20	12	12	5	5	12	6	12	10	10	5	10	10	5	5	5	0	5	5	5	18	5	5	5	5	5	5	10	5	10	5	0	0	

## Exercises of STAT\_2 1392

- 1) Using the underlying data (Download), compute the mean, variance and probability density function of each sets. (Deadline 91/12/10) (20 marks)
- 2) Find different kernels used in calculating probability distribution function and report one of them. Base on kernel you found for previous question to recalculate probability distribution function for the given data. (Deadline 91/12/15) (12 marks)
- 3) Calculate JPFD,  $p(x;y)$ , for few specific points. Plot  $\Delta(\tau_n) = \sum_{x_1, x_2} |p(x_1(t); x_2(t+\tau_n)) - p(x(t)) p(x(t+\tau_n))|$  as a function of  $\tau_n$ . (Deadline 91/12/15) (12 marks)
- 4) Suppose in a box there is a harmonic oscillator. In the random time intervals, we open the door of this box. Derive the probability distribution function of position of mass by using PDF transformation method. (Deadline 91/12/15) (5 marks)
- 5) Prove that a multi-variant PDF in the general form is normalized. (Deadline 91/12/15) (5 marks)
- 6) For three data sets available in the first question calculate  $\sigma_w$  and if possible find  $\tau_x$  in each set. (Deadline 91/12/15) (12 marks)
- 7) Base on Two-Point correlation function, calculate two-point correlation of a 1D data set available in first question for Zero,  $+2\sigma$  and  $-2\sigma$  features. Repeat the same tasks but for 2D data sets (2D-Data). (Deadline 91/12/20) (12 marks)
- 8) Compute the PDF of computer random generator. Make a Gaussian random series using simple and Box Muller methods. Derive analytically Three-point correlation function of  $T(\theta, \phi)$ . (Deadline 91/12/25) (12 marks)
- 9) Calculate the power spectrum of the sun spot data (Download). (Deadline 92/01/20) (10 marks)
- 10) Using about 10000 no. of each data sets given in question 1 and compute the power spectrum for them. Also superimpose each data with some sinusoidal data and compute power spectrum, in your plot show each frequencies embedded in your data. (Deadline 92/01/20) (12 marks)
- 11) Calculate PDF for computer random generator, show the error bars in PDF in your plot. (Deadline 92/01/20) (5 marks)
- 12) Generate a set of Gaussian random numbers with method thought in the class and by calculating the PDF of your data show that it is consistent with Gaussian distribution. (Deadline 92/01/20) (10 marks)
- 13) Generate Non-Gaussian data set with given correlation function (e.g. the shape of correlation function follows a scale invariant behavior  $C \sim \tau^{-\gamma}$ ) and show that three point correlation function of your data is not zero. (Deadline 92/01/20) (10 marks)
- 14) Prove analytically that the real part of Fourier Transform is enough to calculate correlation function. (Deadline 92/01/20) (5 marks)
- 15) For a Langevin equation without drag coefficient, find a relation between temperature and intensity of noise force. (Deadline 92/01/30) (5 marks)

16) Show the equivalence of forward and backward Kramers-Moyal expansion. (Deadline 92/01/30) (5 marks)S

17) Calculate the PDF of a random-walk with Gaussian PDF for each jump. (5 marks)S

18) Show That:

$$P(x, t + \tau | x', t) = \frac{1}{2\sqrt{\pi D^{(2)}(x', t)\tau}} \exp\left\{-\frac{1}{4D^{(2)}(x', t)\tau} [(x - x') - D^{(1)}(x', t)\tau]^2\right\}$$

Show that above result is valid for every  $\tau$ , If  $D^{(1)} = D^{(2)} = \text{constant}$  and  $D^{(3)} = D^{(4)} = 0$  (5 marks)

19) Show that: (5 marks)

$$\lim_{\tau \rightarrow 0} \frac{M_n(x, t, \tau)}{n! \tau} = \begin{cases} D^{(1)}(x, t) & n = 1 \\ D^{(2)}(x, t) & n = 2 \\ 0 & n \geq 3 \end{cases}$$

20) Show That: (5 marks)  $P(x, t + \tau | x', t) =$

$$\frac{1}{2\sqrt{\pi D^{(2)}(x, t)\tau}} \exp\left\{-\frac{\partial D^{(1)}}{\partial x} \tau + \frac{\partial^2 D^{(2)}(x, t)}{\partial x^2} \tau - \frac{1}{4D^{(2)}(x, t)\tau} \left[(x - x') - \left(D^{(1)} - 2\frac{\partial D^{(2)}}{\partial x}\right)\tau\right]^2\right\}$$

Assume that:

$$\mathcal{L}_{FP} = -\frac{\partial D^{(1)}}{\partial x} + \frac{\partial^2 D^{(2)}}{\partial x^2} - \left[D^{(1)} - 2\frac{\partial D^{(2)}}{\partial x}\right] \frac{\partial}{\partial x} + D^{(2)} \frac{\partial^2}{\partial x^2}$$

21) Compute Brownian motion if  $v(t + \Delta t) = v(t) - \epsilon v(t) + \overline{\tau(t)}$  And then calculate numerically: (20 marks)

a)  $P(v(t)) = ?$

b)  $C(v(t), v(t')) = ?$

c)  $\langle v^2 \rangle = ?$

d)  $\langle [x(t + \tau) - x(t)]^2 \rangle = ?$

22) Prove that:  $\langle x^4 \rangle = (\alpha_2 \pm \sigma_2) \langle x^2 \rangle^2 + (\alpha_3 \pm \sigma_3) \cdot \langle x^3 \rangle \sqrt{\langle x^2 \rangle}$ , assume:  $\frac{d}{dt} \langle x^n \rangle = 0$  (5 marks)

23) Calculate analytically:  $\frac{d}{dt} C_x(\tau) = ?$  Start with this relation:  $\frac{d}{dt} \langle [x(t + \tau) - x(t)]^2 \rangle = \dots$  (5 marks)

24) Derive this equation:  $Det' = J^{-2} Det$  (5 marks)

25) Show that  $\bar{D}^i$  is contravariant i.e.  $\bar{D}'^k = \partial x'^k / \partial x^i \bar{D}^i$ . Assume that  $\frac{\partial x'^k}{\partial t} = 0$ . (5 marks)

26) Show that  $\bar{S}^i$  (probability current) is contravariant. (5 marks)

$$\bar{S}^i = \bar{D}^i \bar{P} - \bar{D}^{ij} \frac{\partial \bar{P}}{\partial x_j}$$

27) Show that the divergence of  $\bar{S}^i$  is a scalar. (5 marks)

$$\bar{S}_{;i}^i \equiv \sqrt{Det} \frac{\partial \bar{S}^i}{\partial x^i \sqrt{Det}}$$

28) Plot some deterministic fractals. (10 marks)

29) Calculate fractal dimension with use box counting for 1 and 2 dimensional data (600\*600) (download data from exercise 7). (20 marks)

30) Simulation  $\dot{u} = \varepsilon(t)u$  with assume that:  $\langle \varepsilon(t)\varepsilon(t') \rangle = \gamma D e^{-\gamma t}$  (10 marks)

31) Show that:  $S(\omega) = \omega^{-\beta}$  (5 marks)

32) Use R/S, SWV and Disp method to compute Hurst exponent of 0.2.txt, 0.5.txt and 0.8.txt data sets. (15 marks)