**Name**:

In the name of God

Department of Physics
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# ADVANCED TOPICS IN COSMOLOGY

## Final exam

## (Time allowed: 3 hours)

**NOTE:** All question must be answered. Legibility, good hand-writing and penmanship have 5 additional marks. Please write the answer of each question in separate sheet.

1. Structure formation in the universe. Suppose $\delta(\vec{r}) \equiv \frac{\rho(\vec{r}) - \bar{\rho}}{\bar{\rho}}$ represents density contrast at $\vec{r}$ and the velocity at same place is $\vec{v}(\vec{r}) = H(t)\vec{r} + \vec{v}_p(\vec{r})$. Imagine at $t = t_i$ and $a = a_i$ a density contrast $\delta_i$ with spherical symmetry around an arbitrary position with radius $r_i = a_i\chi$ in flat universe to be constructed. (30 marks).
   **A:** Show that total energy of this region per unite mass at time $t$ if we take $P = w\rho$, is given by:

   $$E = \frac{1}{2}H^2a^2\chi^2\left(1 + \frac{v_p}{Ha\chi}\right)^2 - \frac{1}{2}\chi^2\Omega_0 a^{-(1+3w)}(1 + \delta(a))\frac{H^2}{\Omega_r a^{-4} + \Omega_0 a^{-3(1+w)}}$$

   where $\Omega_0 \equiv \frac{\bar{\rho}_0}{\rho_c^{(0)}}$ and $\Omega_r$ is density of radiation at present epoch.
   **B:** If $w = 0$ show that the condition for collapsing is:

   $$\delta_i > [\Omega_i]^{-1}\left(1 + \frac{v_p(a_i)}{H_i a_i \chi}\right)^2 - 1$$

   where $\delta_i = \delta(a = a_i)$ and $\Omega_i = \Omega(a = a_i)$.
   **C:** In case $w = 0$ show that the maximum value of radius for underlying region is:

   $$r_{max} = D^{-1}[1 + \delta_i]r_i$$

   here $D \equiv (1 + \delta_i) - [\Omega_i]^{-1}\left(1 + \frac{v_p(a_i)}{H_i a_i \chi}\right)^2$.

2. Compute the size of angular diameter distance of an object with diameter in size $D$ at redshift $z$ in flat Universe containing just Dark matter. (10 marks).

3. In the theory of Big-Bang nucleosynthesis, there is a bottle-neck for starting the production of light particles which is so-called high value of entropy. The binding energy for $^2$H, $^3$H, $^3$He and $^4$He are $B = 2.22$meV, $6.92$meV, $7.72$meV and $28.3$meV, respectively. Explain that why the production era for mention elements is around $T \sim 0.07$meV? Prove your explanation mathematically. (20 marks)

4. Alcock-Paczynski (AP) test. Explain and prove the observational parameter based on AP. What is AP equation in the presence of peculiar velocity. What is AP relation in closed and open Universe. (20 marks)

5. CMB frame: An observer recorded fluctuations on the last scattering surface. If this observer has a velocity through a direction $(\ell, b)$ and if the intrinsic temperature of fluctuations is $T_0$:
   **A:** Determine $\Delta T \equiv T(\hat{n}) - T_0(\hat{n})$. (10 marks)
   **B:** According to the latest observation by Planck satellite, the average of temperature is $\bar{T} = 2.7255K$, and in direction $\ell = 264°, b = 48°$ the maximum value of temperature difference is $\Delta T_{max} = 3.4880mK$. Compute the velocity of solar system with respect to the CMB rest frame. (10 marks)