In the name of God

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STATISTICAL FIELD THEORY AND CRITICAL PHENOMENA

Exercise Set 9

(Due Date: 1403/10/30)

1. Wilso-Fisher fixed point:
A: Plot the RG flow for $d < 4$ and $d > 4$ (see Fig. 12.2 chapter 12 Goldenfeld)

2. Perturbative RG up to $\mathcal{O}(u^3)$:
A: According to Kardar's book notation in chapter 5, show that the contribution of $[2] \times [5]$ can be neglected up to $\mathcal{O}(q^2)$. Explain that why are we interested in considering $q \to 0$.
B: Prove the Eq. (5.48) of Kardar's Book.
C: Prove the Eq. (5.54) and (5.55) of Kardar's Book. Then plot the RG flow according to Fig. 5.5.
D: Solve exercises, 1, 4, 6, Kardar's Book, chapter 5.

3. Recursive relation based on dimensional analysis:
According to following Hamiltonian:

$$\beta\mathcal{H} = \int d^d r \left[\frac{K}{2}(\nabla m)^2 + \frac{t}{2}m^2 + u(m^2)^2 + v\sum_{i=1}^{n} m_i^4 \right]$$

here we have n-component vector $m(r) = (m_1(r), m_2(r), ..., m_n(r))$. Now according to RG procedure, and just taking into account rescaling and the re-normalizing, derive the β-function and then derive the correction to scaling exponents. (Hint: this model has been introduced for Cubic symmetry breaking and see sec. 5.8 Cardy's Book)

4. OPE approach to derive recursive relation and scaling exponents:
For the following Hamiltonian and according to OPE formalism, derive the β-function and correction to coupling constant and therefore, find the fixed points and extract the scaling exponents for given fixed points.

$$\beta\mathcal{H} = \int d^d r \left[\frac{K}{2}(\nabla m)^2 + \frac{t}{2}m^2 + \frac{L}{2}(\nabla^2 m)^2 + u(m.m)^2 + v\Pi_{j=1}^{6} m_i \right]$$

5. Solve exercise 12-2 of Goldenfeld's book.

6. Dangerous coupling constant:
Consider a generic model for Free energy density as:

$$f(\phi, \nabla\phi) = \frac{1}{2}r\phi^2(r) + \frac{1}{n}u_n\phi^n(r) - h\phi(r) + \frac{1}{2}|\nabla\phi(r)|^2$$

here $n = 3, 4, 6$ corresponds to percolation, $O(N)$ (including polymer) and tricriticality models, respectively. Show that up to dimensional analysis for $t' = \ell^{x_t} t$, $h' = \ell^{x_h} h$ and $u' = \ell^{x_u} u$,

$$x_t + 2x_\phi = d, \qquad x_t = 2$$

$$x_h + x_\phi = d, \qquad x_h = \frac{d}{2} + 1$$

CONTINUED

$$x_u + n x_\phi = d, \qquad x_u = \frac{d}{2}(2-n) + n$$

It turns out that for $d > \frac{2n}{n-2}$, the u would be relevant. How can manage the following scaling behaviors for $u \to 0.$, namely

$$\lim_{u \to 0} M(t, h, u)$$

for two cases $M(t, 0, 0) \propto t^{\beta}$ and $M(0, h, 0) \propto h^{1/\delta}$. Show that

$$\beta = -\frac{x_h - d}{x_t} + \frac{x_u}{2x_t}$$

and

$$\frac{1}{\delta} = -\frac{x_h - d}{x_h} + \frac{1}{3}\left(\frac{x_u}{x_h}\right)$$

Good luck, Movahed