In the name of God

# Department of Physics
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## STATISTICAL FIELD THEORY AND CRITICAL PHENOMENA

### Exercise Set 8

### (Due Date: 1403/10/10)

**1.** Beta Function: Suppose that

$$\beta\mathcal{H}[m] = \int d^d r \left[\frac{t}{2}m^2 + \frac{K}{2}\left(\nabla m\right)^2 + \frac{L}{2}\left(\nabla^2 m\right)^2 + ...\right] + u\int d^d r m^4$$

after computing the Hamiltonian up to $\mathcal{O}(u^2)$, the correction in coupling constants read as:

$$\bar{K} = K - u^2 A$$
$$\bar{t} = t + 4(n+2)u\int_{\Lambda/\ell}^{\Lambda}\frac{d^d q}{(2\pi)^d}\frac{1}{t+kq^2+Lq^4} - u^2 B$$
$$\bar{u} = u - 4(n+8)u^2\int_{\Lambda/\ell}^{\Lambda}\frac{d^d q}{(2\pi)^d}\frac{1}{t+kq^2+Lq^4}$$

here $A$ and $B$ are constants. Now based on RG, namely $q = \ell^{-d}q'$, $m = zm'$, show that $K_\ell = \ell^{-d-4}z^2\bar{K}$, $t' = \ell^{-4}z^2\bar{t}$ and $u' = \ell^{-3d}z^4\bar{u}$. Also determine $L'$. By linearizing the recursive relation, determine the beta function. Find the fixed points and draw the RG-flow. (Hint: See section 5.7 of Kardar's Book)

**2.** Irrelevance coupling constants. Suppose that

$$\beta\mathcal{H}[m] = \int d^d r \left[\frac{t}{2}m^2 + \frac{K}{2}\left(\nabla m\right)^2 + \frac{L}{2}\left(\nabla^2 m\right)^2 + ...\right] + \int d^d r \left[um^4 + vm^2(\nabla m)^2 + ... + u_6 m^6 + ... + u_8 m^8\right]$$

Derive the renormalized coupling constants by RG. Around Gaussian fixed point, determine the scaling exponents of coupling constants and accordingly determine the irrelevant coupling constants. (Hint: See section 5.8 of Kardar's Book)

Good luck, Movahed