In the name of God

Department of Physics
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STATISTICAL FIELD THEORY AND CRITICAL PHENOMENA

Exercise Set 6

(Due Date: 1403/09/01)

1. Fluctuations around Tricritical point: Exercise 3.4 Kardar's Book.

2. Spin wave: Exercise 3.1 Kardar's Book.

3. Random magnetic field: a model for presence of impurities in matter. Suppose that we have

$$L[\phi] = \int d^d r \left[\frac{t}{2}\phi(r)^2 + u\phi(r)^4 + \frac{K}{2}(\nabla\phi(r))^2 - h(r)\phi(r) \right]$$

where $\phi(r)$ and $h(r)$ are scalar field and $u > 0$. Suppose that $h(r)$ comes from Gaussian random field, namely

$$\langle h(r) \rangle = 0$$

and

$$\langle h(r)h(r') \rangle = \delta_d(r - r')$$

and

$$p(h(r)) = \exp\left(-\frac{h(r)^2}{2} \right)$$

A: Calculate free energy according to saddle point approximation.
B: Suppose $\phi = \phi_0 + \psi$ and in Gaussian approximation, compute the correction on zeroth order approximation of free energy.
C: Determine the discontinuity on heat capacity at critical point. (Hint: see the exercise 3.6 Kardar's Book)

4. RG in coordinate space:
A: Exercise 6.2 of Statistical physics of fields written by M. Kardar.
B: Exercise 6.3 of Statistical physics of fields written by M. Kardar.

5. Widom scaling hypothesis: Exercise 4.1 of Statistical physics of fields written by M. Kardar.

6. Exercises no. 14.1, 14.2 of Statistical Mechanics written by Pathria and Beale.

7. Exercises no. 9.1, 9.2 and 9.3 of Goldenfeld.

Good luck, Movahed