In the name of God

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STATISTICAL FIELD THEORY AND CRITICAL PHENOMENA

Exercise Set 4

(Due Date: 1403/08/07)

1. Exercises no. 1, 2, 3, 7 of chapter 2, Cardy.

2. According to Saddle point approximation, show that Mean-Field theory would be exact for $d \to \infty$.

3. To answer that what the relation between T_c and T_c^{MF}, we have started by following Hamiltonian:

$$\mathcal{H} = \int d^d r \ \left[t + 3uM^2\right] s^2(r)$$

also we know that $[\xi^{-2}] \sim [t + 3uM^2]$, and

$$M^2 \equiv \langle s^2(r)\rangle = G(0) = \int \frac{d^d k}{(2\pi)^d} \frac{\ell^2}{k^2 + \xi^{-2}}$$

in addition considering a UV cutoff, show that:

$$\lim_{d\to\infty} T_c^{MF} = T_c$$

4. According to entropy as $S = -K_B \sum_s P(s) \ln P(s)$, where $P(s)$ is the probability of finding a spin whose value is s and by using $F = E - TS$, compute the $\langle s\rangle$. (Hint: suppose that $s = \pm 1$ and $P(s = +1) + P(s = -1) = 1$ and $\langle s\rangle = \sum_s sP(s)$, $E = \langle \mathcal{H}\rangle_{mean-field}$, where $\mathcal{H} = -J \sum s_i s_j - H \sum s_i$)

5. Exercises no. 1.3, 1.6, Kardar

6. The fluctuations in a given observable quantities in principle is given by proper derivative of corresponding partition function with dual parameters. (Remember that in a canonical ensemble the fluctuation in energy is $\sigma^2_{\mathcal{H}} = \langle(\mathcal{H} - \langle\mathcal{H}\rangle)^2\rangle = -\frac{\partial^2}{\partial\beta^2} \ln Z$). Accordingly, for the grand-canonical ensemble, we suppose to have $\sigma^2_N = \langle(N - \langle N\rangle)^2\rangle$. Also the weighted TPCF of number density is given by $G(r - r') = \langle\rho(r) - \rho(r')\rangle - \langle\rho(r)\rangle\langle\rho(r')\rangle$, since the Number of particle is $N = \int dr\rho(r)$, find the relation between number fluctuation in grand-canonical ensemble. Interpret the physical meaning of your result.

7. Periodic and open boundary 1D Ising model: Show that at thermodynamical limit, these two models lead to similar results.

Good luck, Movahed