

In the name of God

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STATISTICAL FIELD THEORY AND CRITICAL PHENOMENA

Exercise Set 3

(Due Date: 1403/08/01)

1. Response function: Suppose that  $\mathcal{H} = \bar{\mathcal{H}}(\{S\}) + \sum_i H_i S_i$ 
  - A: Show that  $G(i, j) = \frac{1}{\beta} \frac{\partial}{\partial H_i} \left( \frac{1}{Z\beta} \frac{\partial Z}{\partial H_j} \right)$ . Explain the meaning of this quantity.
  - B: Write down the  $\sum_{ij} G(i, j)$  using partition function.
  - C: Suppose that for Homogeneous case  $G(R) = \frac{1}{R^\sigma} f(R/\xi)$ . Derive the susceptibility in  $d$ -Dimension.
2. Mean field theory: Show that  $F^{\text{mean-field}} \geq F$  for Ising model. What about  $T_c^{\text{mean-field}}$  and  $T_c$ .
3. Mean field theory: Incorporating the correction up to  $\mathcal{O}(\delta s^3)$ , derive the effective magnetic field at location of  $s_i$ .
4. Show that the coefficient of  $M^2$  in thermodynamical free energy,  $f = a + btM^2 + cM^4 + \dots$  is related to correlation length scale ( $\xi$ ), namely  $bt \sim \xi^{-2}$ . What is the meaning of that?

Good luck, Movahed

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