In the name of God

Department of Physics
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CRITICAL PHENOMENA

Exercise Set 5

(Due Date: 1401/09/15)

1. Exercises no. 1.3, 1.6, Kardar

2. The fluctuations in a given observable quantities in principle is given by proper derivative of corresponding partition function with dual parameters. (Remember that in a canonical ensemble the fluctuation in energy is $\sigma_{\mathcal{H}}^2 = \langle(\mathcal{H}-\langle\mathcal{H}\rangle)^2\rangle = -\frac{\partial^2}{\partial\beta^2}\ln Z$). Accordingly, for the grand-canonical ensemble, we suppose to have $\sigma_N^2 = \langle(N-\langle N\rangle)^2\rangle$. Also the weighted TPCF of number density is given by $G(r-r') = \langle\rho(r)-\rho(r')\rangle-\langle\rho(r)\rangle\langle\rho(r')\rangle$, since the Number of particle is $N = \int dr\rho(r)$, find the relation between number fluctuation in grand-canonical ensemble. Interpret the physical meaning of your result.

3. Tricritical phase transition: According to the sixth order Landau theory, suppose that

$$\mathcal{L} = -hm + \frac{1}{2}am + \frac{1}{4}bm^4 + \frac{1}{6}cm^6$$

where $a = a(t)$, $b = b(t)$ and $c = c(t)$. In order to have physical value for m, deduce the necessary properties of a, b, c and their temperature dependancies. Finally for proper value of multipliers determine the m 's for them the landau free energy has extremums.

Good luck, Movahed