In the name of God

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MODERN COSMOLOGY

Exercise Set 4

(Due Date: 1404/02/20)

1. For open ($\Omega^0_{total} < 0$), closed ($\Omega^0_{total} > 0$) and flat ($\Omega^0_{total} = 0$) universes and in each case use $w_\lambda = -1$ and $w_\lambda = -2$ and $w_\lambda = -0.1$, compute and plot following parts:

A : plot H as a function of a, z and t.

B : plot $\ddot{a}/a$ as a function of a, z and t.

C : plot ρ_m, ρ_r and ρ_λ as a function of a, z and t. Determine the dominant epoch of each components.

D : plot deceleration parameter as a function of redshift. Determine the value of redshift which corresponds to accelerating era of universe.

E : Comoving length scale as a function of a and z. (Hint: $\chi_0 - \chi = c\int_t^{t_0} \frac{dt'}{a(t')}$).

F : Look back time (cosmic age), $t(z)$. Also plot t as a function of χ.

G : Angular diameter distant, d_A.

H : Luminosity distance, d_l. Use data of Gold sample and then compute distance modulus, $\mu \equiv m - M$ and compare your results for $\Omega^0_m = 0.31$ and $\Omega^0_\lambda = 0.68$ and $w_\lambda = -1.0$. What happens if we have $w_\lambda > -1$ or $w_\lambda < -1$ (i.e. equation of state of dark energy), Show them in the same plot.

I : Comoving volume element Luminosity distance, d_l. Use data of Gold sample and then compute distance modulus, $\mu \equiv m - M$ and compare your results for $\Omega^0_m = 0.31$ and $\Omega^0_\lambda = 0.68$ and $w_\lambda = -1.0$. What happens if we have $w_\lambda > -1$ or $w_\lambda < -1$ (i.e. equation of state of dark energy)Show them in the same plot.

J : Compute maximum visible age of astronomical object as a function of redshift for $\Omega^0_m = 0.31$ and $\Omega^0_\lambda = 0.68$ and $w_\lambda = -1.0$. What happens if we have $w_\lambda > -1$ or $w_\lambda < -1$ (i.e. equation of state of dark energy)Show them in the same plot.

K : Plot the traveling path of a photon emitted from the horizon of an observer at the big-bang epoch and moves through the observer. Suppose that $\Omega^0_m = 0.31$ and $\Omega^0_\lambda = 0.68$ and $w_\lambda = -1.0$. What happens if we have $w_\lambda > -1$ or $w_\lambda < -1$ (i.e. equation of state of dark energy)Show them in the same plot.

L : Compare the Hubble velocity and peculiar velocity for small redshift.

M : Investigate Alcock-Paczynski quantity and compute it in terms of redshift. Suppose that $\Omega^0_m = 0.31$ and $\Omega^0_\lambda = 0.68$ and $w_\lambda = -1.0$. What happens if we have $w_\lambda > -1$ or $w_\lambda < -1$ (i.e. equation of state of dark energy)Show them in the same plot.

N : Compute CMB shift parameter. Suppose that $\Omega^0_m = 0.31$ and $\Omega^0_\lambda = 0.68$ and $w_\lambda = -1.0$ and $v_s = \frac{c}{\sqrt{3}}$. What happens if we have $w_\lambda > -1$ or $w_\lambda < -1$ (i.e. equation of state of dark energy)Show them in the same plot.

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O : Compute Baryon Acoustic oscillation quantity, β.

$$\beta = \left[\frac{H(z=0.2)d_l^2(z=0.35)0.35(1+0.2)^2}{H(z=0.35)d_l^2(z=0.2)0.2(1+0.35)^2}\right]^{1/3}$$

Suppose that $\Omega_m^0 = 0.31$ and $\Omega_\lambda^0 = 0.68$ and $w_\lambda = -1.0$ and $v_s = \frac{c}{\sqrt{3}}$. What happens if we have $w_\lambda > -1$ or $w_\lambda < -1$ (i.e. equation of state of dark energy)Show them in the same plot.

P : Suppose we have a dynamical dark energy in the universe with $w_\lambda = w_0 a^{-\alpha}$ and $0 \leq \alpha \leq 1$. Do all

Q : In the plane of $(\Omega_m^0, \Omega_\lambda^0)$ and supposing $w_\lambda = -1$, plot the constant curve for $t_0 = 13.86$Gyr. Show the region that big-bang is not allowed.

R : Optical depth: The probability to intersect an object with redshift smaller than z called optical depth, i.e.

$$\tau(z) = \pi r_*^2 c \int_0^z \frac{n(z')dz'}{H(z')(1+z')}$$

imagine that $n(z) = n_0(1+z)^3$, $n_0 \sim 0.02h^3 Mpc^{-3}$, $r_* \sim 10h^{-1}Kpc$, compute and plot optical depth. At which redshift our universe will be opaque based on this cosmological objects.

2. Suppose that the number density of astronomical objects is constant, $n = cts$, compute the number of astronomical object as a function of redshift. In the case of $n = \exp(-z)$ repeat your calculation.

3. Suppose that we have matter dominant era, according to $\left(\frac{da}{dt}\right)^2 = \frac{8\pi G}{3}a^2\rho_m - k$, where k determines geometry of universe, solve analytically $a(t)$ as a function of t. (Hint: define an auxiliary variable let say x and then calculate $a(x)$ and $t(x)$)

Good luck, Movahed