In the name of God

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MODERN COSMOLOGY

Exercise Set 2

(Due Date: 1404/01/20)

1. By using the Schwarzschild metric for a static and homogeneous universe, show that the spatial part of displacement $d\ell^2 = a(t)^2[f(r)dr^2 + r^2 d\Omega^2]$ such that $f(r) = 1/(1 - Kr^2)$. Here K represents the curvature. (Hint: Use the Ricci scaler approach).

2. Gravitational Redshift: According to the Schwarzschild metric, for static, spherically symmetric around a massive object, use the following line element as:

$$c^2 d\tau^2 = -A(r)dt^2 + B(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

derive $A(r)$ and $B(r)$. Now consider $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and $h_{\mu\nu}$ is due to massive object, and by using null geodesic equation, derive

$$\frac{\nu_{obs}}{\nu_{sour.}} = \left(\frac{1 - \frac{2MG}{r_{sour}c^2}}{1 - \frac{2MG}{r_{obs}c^2}} \right)^{1/2}$$

where r_{sour} and r_{sour} are the position vectors of observer and source, respectively.

3. For massive object and by using the four-dimensional energy momentum vector and the geodesic equation in absent external force for expanding universe, the scale factor dependency of momentum.

4. Derive the proper volume of 2D and 3D spaces for general curvature.

5. Energy-momentum conservation in general relativity. Using the general form of $T_{\mu\nu} = \rho u_\mu u_\nu + P\gamma_{\mu\nu} + q_\mu u_\nu + \pi_{\mu\nu}$, derive continuity and Euler equations. Where $\rho = T_{\mu\nu}u^\mu u^\nu$, $P = T_{\mu\nu}\gamma^{\mu\nu}/3$, $q^\mu = -T_{\alpha\beta}u^\alpha\gamma^{\beta\mu}$, $\pi_{\mu\nu}$ is anisotropic pressure tensor, $\gamma_{\mu\nu}$ is 3D Reimannian metric. Use the signature $(-,+,+,+)$.

6. Using Einstein equation as $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}$, and $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$, where $g_{\mu\nu} = (c^2, -\frac{a^2(t)}{1-kr^2}, -a^2(t)r^2, -a^2(t)r^2\sin^2(\theta))$. For Ideal flow, derive equation evolution for scale factor. Do the same if the cosmological constant is added in mentioned equation.

7. Solve all exercises of chapter 2, Modern Cosmology Book written by S. Dodelson and F. Schmidt, 2021 edition.

Good luck, Movahed