بسم الله الرحمن الرحیم

جلسه ۱۴/۱۲/۱۴۰۲

Olbers's Paradox 1823 A.D.

پارادکس اولبرس

There must be No dark sky!

آسمان شب وجود ندارد.

☆ It was supposed that our Universe has following properties:

- Homogeneous — همگن
- Isotrop — همسانگرد
- Static — ایستا → (Steady State)
- Infinite — بی‌نهایت
  - Time → age
  - Size

Energy Flux at location of an observer

هدف محاسبه این کمیت است →

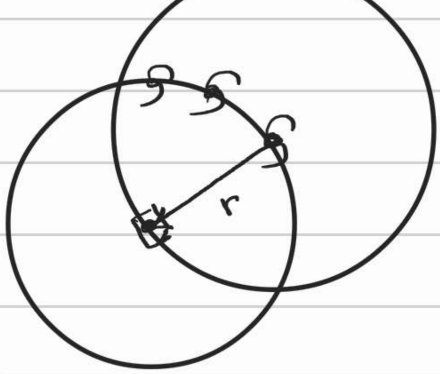
$$dJ \equiv \frac{d^2E}{dA\,dt} = B\,dN$$

dN ← No. of source

B ← Surface Brightness

$$B = \frac{h\nu}{At} = \frac{L}{A}$$

L → Luminosity



$$dN = 4\pi r^2\,dr\,n(r)$$

$n(r)$ → فرض $n = ck$

ایستا یا پویا

Olbers Paradox 1823 A.D

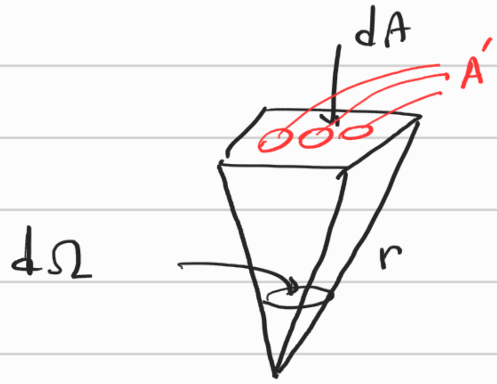
۱) نیوتن فرض کرده بود که جهان تا بی نهایت گسترش یافته - یکنواخت - ایستا
۲) عالم شفاف نیست
۳) سرعت نور بی نهایت نیست
۴) سن عالم برای اینکه فرصت داشته باشیم همه ستاره ها را ببینیم برابر است با $t : 10^{23} y$
۵) جهان در حال انبساط است

$$dJ = B\,dN$$
$$= \frac{L}{A}\, 4\pi r^2\, dr\, n$$
$$dJ = \frac{L\, \cancel{4\pi r^2}\, dr\, n}{\cancel{4\pi r^2}} = L\, n\, dr$$

number density of source

$$J = \int_0^\infty dJ = \int_0^\infty L \times n \times dr$$
$$J = L \times n \int_0^\infty dr \rightarrow \infty$$

☆ a Solution



$$dA = r^2\, d\Omega$$
$$A'\, dN = r^2\, d\Omega$$

$4\pi r^2 dr\, n(r)$

$$\rightarrow d\Omega = \frac{A'\, dN}{r^2}$$

cts

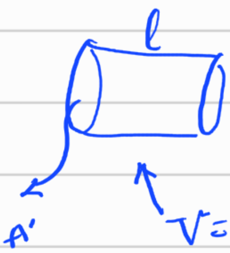
$$4\pi = \int d\Omega = \int_0^{R_{max}} 4\pi \cancel{r^2}\, dr\, \frac{n(r)\, A'}{\cancel{r^2}}$$

$$\cancel{4\pi} = \cancel{4\pi}\, n\, A'\, R_{max} \rightarrow \boxed{R_{max} = \frac{1}{nA'}}$$

$$J = \int dJ = \int_0^{\cancel{\infty} \rightarrow R_{max}} L \times n \times dr = L \times n \times R_{max}$$
$$= \frac{L \times \cancel{n}}{A' \times \cancel{n}} = \frac{L}{A'} = \frac{L_\odot}{A_\odot} \neq 0$$

☆ Arrival time : The time, it takes for light to reach the observer.

$$\underbrace{A' l n = 1} \rightarrow l = \frac{1}{nA'} = c\tau_c \qquad \boxed{\tau_c = \frac{l}{c}}$$



مدت زمان متوسط مسافرت یک فوتون قبل برخورد با جسم

$V = lA'$ — $V \times n = 1$

$\tau_c = \langle \tau \rangle = \int d\tau\, \tau P(\tau)$

$l_c = \langle l \rangle = \int dl\, l\, P(l)$

$$\tau_c = \frac{l}{c} = \frac{1}{nA_\odot c} = \frac{1}{n\pi R_\odot^2 c} = \frac{1}{\pi R_\odot^2 c} \cdot \frac{\frac{4\pi}{3} r_*^3}{N}$$

$$N \equiv \frac{\text{total Mass}}{M_\odot} = \frac{M_{total}}{M_\odot} \longrightarrow \boxed{\tau_c \sim 10^{23}\ yr}$$

$\tau_c > t_0 \sim t_H$

$10^{23} > 10^{10}$

$n = ?$

$\langle l \rangle = l_c$

☆ تابع توزیع مسافت طی شده توسط نور پیش از برخورد در فاصله l و $l+dl$ — $P(l) = ?$

تابع توزیع فاصله زمانی سپری شده پیش از برخورد: در زمان τ و $\tau + d\tau$ — $P(\tau) = ?$

$\langle \tau \rangle = \tau_c$

Recall: Poisson Distribution $$P(n) = \frac{e^{-\lambda} \lambda^n}{n!}$$

احتمال رخ دادن n تعداد حالت مطلوب: تعداد شمارش

$$\lambda \equiv Np \qquad \boxed{N \to \infty}$$

تعداد کل — احتمال در بار

$$P_{Binomial}(n) = \binom{N}{n} p^n (1-p)^{N-n}$$

$$\lim_{\substack{N\to\infty \\ p\to 0 \\ Np\to\lambda}} P_{Binomial} = P_{Poisson} = \frac{e^{-\lambda}\lambda^n}{n!}$$

$\langle n\rangle = \lambda$

$\langle (n-\langle n\rangle)^2\rangle = \lambda$

احتمال اینکه حداکثر تا فاصله l تعداد n تا برخورد داشته باشیم

$$P(n,l) = \frac{e^{-(\Gamma l)}(\Gamma l)^n}{n!}$$

$\Gamma \equiv \frac{1}{l_c}$

$$\quad = \frac{e^{-l/l_c}(l/l_c)^n}{n!}$$

$$P(n=0, l) = \frac{e^{-l/l_c}(l/l_c)^0}{0!} = e^{-l/l_c}$$

← احتمال اینکه من تا فاصله l هیچی دریافت نکنم

$$\underbrace{P(n\geq 1, l)} = 1 - e^{-l/l_c}$$

← احتمال دریافت فوتون تا فاصله $\underline{\underline{l}}$

$$\underbrace{p(l)\,\Delta l} = \frac{dP(n\geq 1, l)}{dl}\Delta l = \frac{e^{-l/l_c}}{\underbrace{l_c}}\Delta l$$

$$p(l) = \frac{e^{-l/l_c}}{l_c} = \quad \text{PDF}$$

$$1 = \int_0^\infty dl\, p(l)$$

$$P(r_*) = \int_0^{r_*} dl\, \frac{e^{-l/l_c}}{l_c} = 1 - e^{-\frac{r_*}{l_c}}$$

To have Dark Sky $\quad P(r_*) \to 0 \quad \to \quad \dfrac{r_*}{l_c} \to 0 \begin{cases} r_* \to 0 \\ l_c \to \infty \end{cases}$

$$l_c = \tau_c C$$

$$\frac{r_*}{l_c} = \frac{t_0 \cancel{c}}{\tau_c \cancel{c}} = \frac{\overset{\sim 10^{10}\,yr}{t_0}}{\tau_c} \sim 10^{-13} \to 0$$

We have Dark Sky

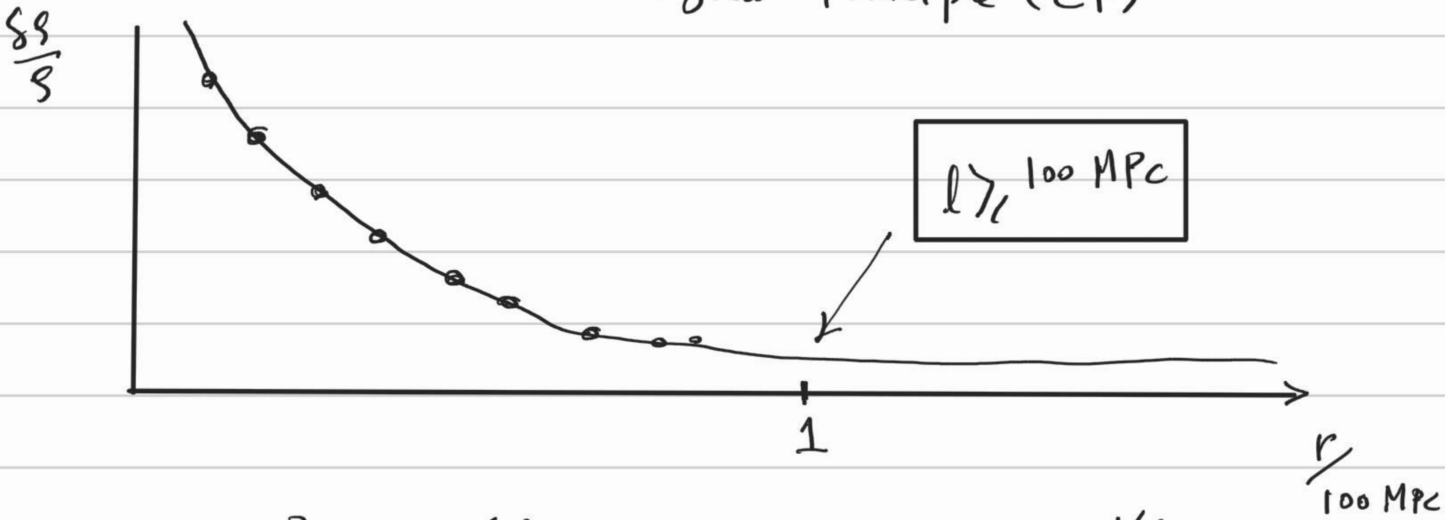
در نظر گرفتن انبساط کیهان همچنین منجر به تاریک تر شدن هم می‌شود

Z: Redshift

Chapter 2: The Expanding Universe.

$\underbrace{\text{Homogeniety and Isotropy}}_{\text{Cosmological Principle (CP)}}$ assumptions

فرض همگنی و همسانگردی



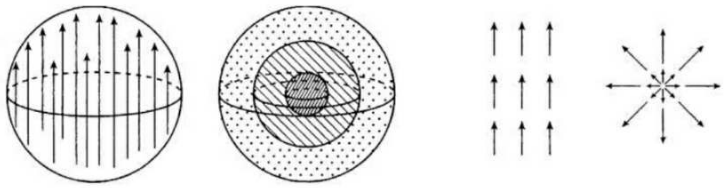
$r = 3\,m \qquad \dfrac{\delta\rho}{\rho} = \dfrac{\rho - \langle\rho\rangle}{\langle\rho\rangle}, \quad \rho \sim 100\ Kg/m^3$

$r = 1\,Au \qquad \rho \sim 4 \times 10^{-5}\ Kg/m^3$

$r = 100\ Mpc$ $\rho \sim 5\ Proton/m^3$

اصول کیهانشناختی

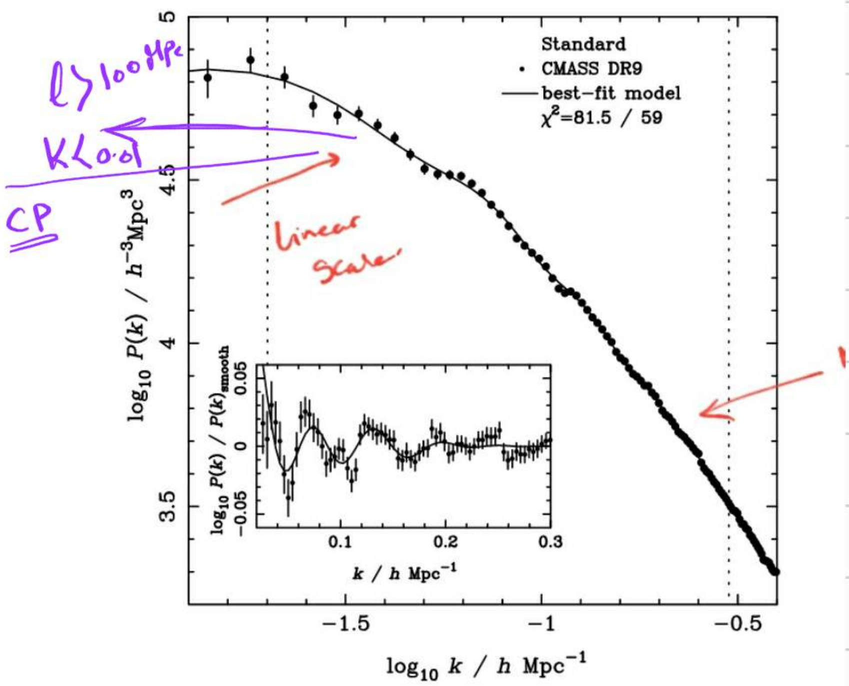
۱) هیچ مرجع مرجحی وجود ندارد
۲) همگنی Homogeneity
۳) همسانگردی Isotropy



$L \geq 100\,Mpc$ $1Mpc = 3.3\times10^{6}\,light\ year = 3.09\times10^{24}\,cm$

فاصله بین کهشانها $:\ 5\,Mpc$

مقیاس عالم قابل مشاهده $:\ 14000\,Mpc$



* The Foundation of Cosmology
- Kinematic
- Dynamics

☆ Kinematics includes : Coordinates مختصات
Metric سنجه
Geodesics ژئودزیک‌ها

☆ Dynamics includes : Einstein Equa

$$\boxed{G_{\mu\nu} = 8\pi G\, T_{\mu\nu}}$$

① A brief about Generalized Coordinates Systems

{ My goal is derivation of a Systematic Framework to determine value of Infinitesimal displacement from different Reference frames }

* What is metric ?

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu$$

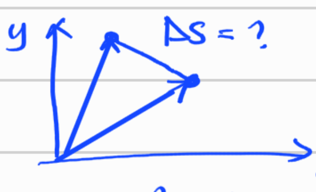
Ⓐ Metric Transforms observer (observational) associated coordinates to invariant Quantity.

Ⓑ GR: We consider Gravity as an Intrinsic Property of Metric

$g_{\mu\nu}$
- Information about geometry
- Gravity

Cartesian

Ex.: 2D (x, y)



$$\Delta s^2 = \Delta x^2 + \Delta y^2 \longrightarrow \Delta s^2 = g_{ij}\, \Delta x^i \Delta x^j \longrightarrow g_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

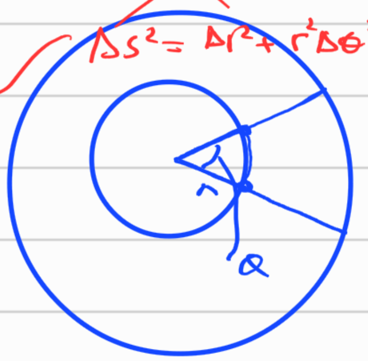
Polar coordinates $(x, y) \longrightarrow (r, \theta)$

↓

$$\Delta S^2 = \Delta x^2 + \Delta y^2 \longrightarrow \Delta S^2 = \cancel{\Delta r^2} + \Delta\theta^2$$

$$g_{ij}^{\text{Polar}} = \begin{pmatrix} 1 & 0 \\ 0 & r^2 \end{pmatrix}$$

$$\Delta S^2 = \Delta r^2 + r^2 \Delta\theta^2$$



In Cartesian Coordinates: (x, y, z)

(x^1, x^2, x^3)

$$\left\{ \begin{array}{l} x(q_1, q_2, q_3) \\ y(q_1, q_2, q_3) \\ z(q_1, q_2, q_3) \end{array} \right. \quad \& \quad \begin{array}{l} q_1(x, y, z) \\ q_2(x, y, z) \\ q_3(x, y, z) \end{array}$$

(x, y, z)

$\vec{e}_x, \vec{e}_y, \vec{e}_z$

$$\left. \begin{array}{l} \hat{e}_x = \dfrac{\vec{e}_x}{|\vec{e}_x|} = \hat{i} \\[2ex] \hat{e}_y = \dfrac{\vec{e}_y}{|\vec{e}_y|} = \hat{j} \\[2ex] \hat{e}_z = \dfrac{\vec{e}_z}{|\vec{e}_z|} = \hat{k} \end{array} \right\}$$

$$\vec{e}_{q_1}, \vec{e}_{q_2}, \vec{e}_{q_3} \longrightarrow \text{In General Case} \left\{ \begin{array}{l} |\vec{e}_{q_i}| \neq 1 \\ \vec{e}_{q_i} \cdot \vec{e}_{q_j} \neq \delta_{ij} \end{array} \right.$$

Position Vector $\equiv \vec{S} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z$

$$d\vec{s} = dx\,\hat{e}_x + dy\,\hat{e}_y + dz\,\hat{e}_z$$

$$\begin{cases} dx = \dfrac{\partial x}{\partial q^1} dq^1 + \dfrac{\partial x}{\partial q^2} dq^2 + \dfrac{\partial x}{\partial q^3} dq^3 \\[2ex] dy = \dfrac{\partial y}{\partial q^1} dq^1 + \dfrac{\partial y}{\partial q^2} dq^2 + \dfrac{\partial y}{\partial q^3} dq^3 \\[2ex] dz = \dfrac{\partial z}{\partial q^1} dq^1 + \dfrac{\partial z}{\partial q^2} dq^2 + \dfrac{\partial z}{\partial q^3} dq^3 \end{cases}$$

$$\underbrace{d\vec{s}} = \frac{\partial \vec{s}}{\partial q^i} dq^i = \underbrace{\frac{\partial \vec{s}}{\partial q^1}} dq^1 + \underbrace{\frac{\partial \vec{s}}{\partial q^2}} dq^2 + \underbrace{\frac{\partial \vec{s}}{\partial q^3}} dq^3$$

$$dx\,\hat{e}_x + dy\,\hat{e}_y + dz\,\hat{e}_z = \vec{e}_{q_1}\, dq^1 + \vec{e}_{q_2}\, dq^2 + \vec{e}_{q_3}\, dq^3$$

$$\boxed{\vec{e}_{q_i} \equiv \frac{\partial \vec{s}}{\partial q^i}} \qquad \boxed{|\vec{e}_{q_i}| \equiv h_i}$$

$$\vec{e}_{q_i} = h_i\, \hat{e}_{q_i}$$ Unit Vector

$$\boxed{d\vec{s} = h_i\, \hat{e}_{q_i}\, \underbrace{dq^i}_{\text{Contravariant}}}$$

$$\boxed{\vec{e}_i = \frac{\partial \vec{S}}{\partial q^i}}$$: Covariant basis

(tangent basis)

They are tangent to Coordinate span

$$\boxed{\vec{e}^{\,i} = \vec{\nabla} q^i}$$: ContraVariant Basis.

(Cotangent Basis)

They are orthogonal to the Coordinate surface

$$\bar{e}'_i = \frac{\partial x^j}{\partial x'^i} \vec{e}_j$$

$$\bar{e}'^i = \frac{\partial x'^i}{\partial x^j} e^j$$

$$\boxed{\vec{A} = A^i \vec{e}_i = A_i \vec{e}^{\,i}}$$

$$A \cdot A = A^i A_i = g_{ij} A^i A^j$$

$$= A^i g_{ij} A^j = A^i A_i$$