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جلسه ۵، ۱۲، ۱۴۰۳

* A brief on Relativity

① Transformation: A systematic approach to examine a phenomenon by observers in different Reference frames:

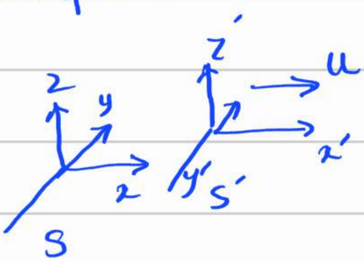
② Relativity ⟶ Exotic theory

- ⟶ Kenematics سینتیک
  - ⟶ Coordinates, Metric, Geodesic . . . –
- ⟶ Dynamics دینامیک
  - ⟶ Einestein Equation

③ Transformation: $\underbrace{\text{Galilean and Newton Prespective}}_{\text{Classical Relativity}}$

④ Alternative Form: Einstein Prescription

⑤ Galilean Coordinate Transformation

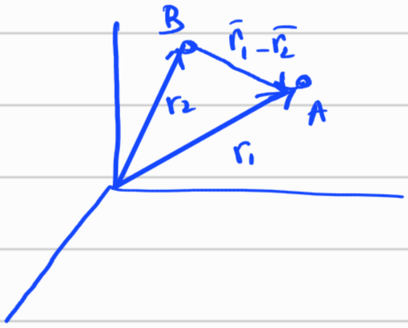


$$x' = x - ut$$
$$y' = y \quad z' = z, \quad t = t'$$

⑥ { ☆ Galilean Velocity Transformation
☆ " acceleration "

⑦ Intraction between Two Particles from Two

Inertial Reference frames



$$\vec{F}_{AB} = -\vec{\nabla}\phi(r_{AB}) \longrightarrow \vec{F}'_{AB} = ?$$

$$\vec{F}_{AB} = \vec{F}'_{AB} \rightarrow \text{According to}$$

Galilean Transformation

⑧ Some Consequences of Classical Relativity:

☆ No Preferred Inertial observer

☆ Impossibility of discrimination between

Inertial Frames by Mechanical law

☆ Impossibility of determining absolute Velocity of Inertial Frames.

$\longrightarrow$ { Newton's law is invariant between Inertial reference frames. }

⑨ What about Maxwell Equation (Electro-magnetism). ?

Wave Equation $\boxed{\text{Conductivity Coefficient}}$ $g = 0$

$$\nabla^2 \vec{E}(\vec{r}) - \epsilon\mu \frac{\partial^2 \vec{E}(\vec{r})}{\partial t^2} - g\mu \frac{\partial E}{\partial t} = 0$$

Vacuum $\rightarrow J=0$

$\rightarrow \rho=0$

No source.

Z-direction for Propagation

$$\frac{d^2E}{dz^2}(z,t) - \left(\frac{\omega}{c}\right)^2 E = 0$$

$$C \equiv \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$E(r,t) = E_0 e^{-i(\omega t \mp kz)}$$

$$\boxed{\nabla^2 \phi - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial x^2} = -\rho/\epsilon_0}$$

$$K \equiv \frac{\omega}{c}$$

$$\left\{\begin{array}{ll} \omega t - kz & +z \\ \omega t + kz & -z \end{array}\right\}$$

(10) **Covariant Form of Physic's laws ?**

(11) Ether ? Reference frame.

Absolute.

(12) Einstein's Postulates :

(The Principle of Relativity)
① The laws of physics are the same in all Inertial Reference frames.

② The Principle of Constancy of Speed of Light in all inertial reference frame.

(13) Consequences of Einstein's Postulates

A: Relativity of Time. : Time dilation

(B) Relativity of Length : Length Contraction

(C) Relativistic Velocity Additivity.

$$\Delta t = \frac{\Delta t_0}{\underbrace{\sqrt{1-u^2/c^2}}_{0< \ <1}} \longrightarrow \Delta t > \Delta t_0$$

$\cancel{c \pm u}$ $\longrightarrow$ $\boxed{v = \dfrac{v' + u}{1 + v'u/c^2}}$

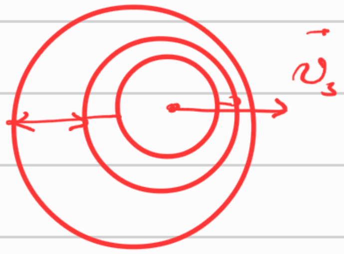
$$l = l_0\sqrt{1-u^2/c^2} \longrightarrow l < l_0$$

(14) Relativistic Doppler Effect:

Non-Relativistic Doppler Effect:

$$f_{obs} = f_s \, \frac{v^2 - \vec{v}\cdot\vec{v}_{obs}}{v^2 - \vec{v}\cdot\vec{v}_s}$$

$$v = \lambda f$$



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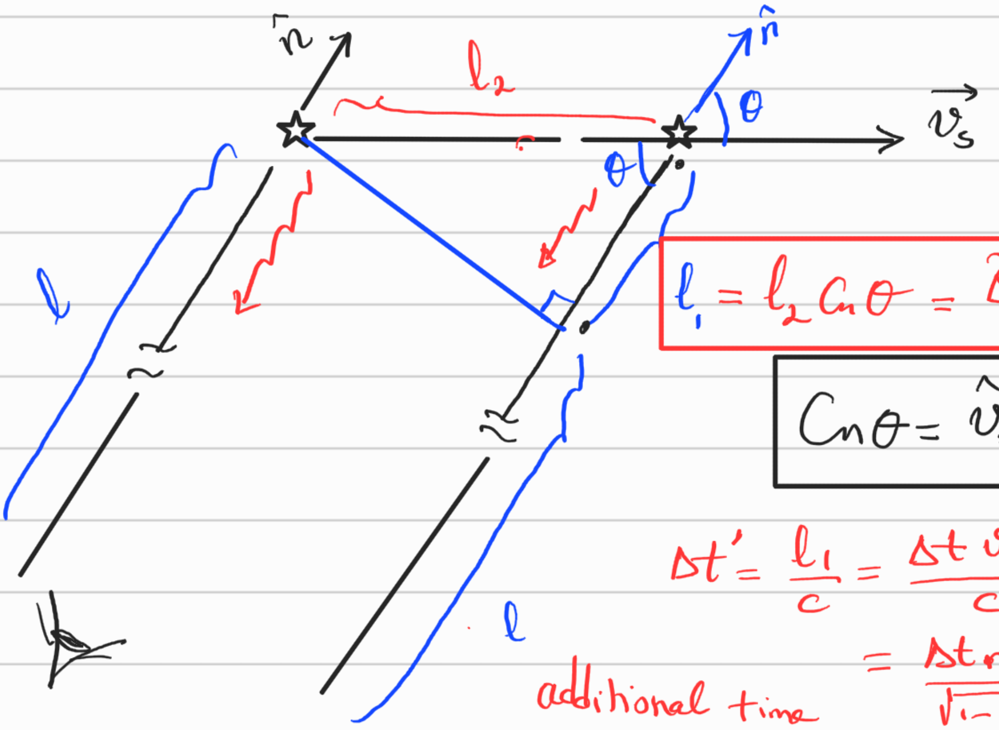
Cosmological Redshift

Gravitational Redshift

What about Relativistic Doppler Effect?



Time interval between two Pulses

$$\boxed{l_1 = l_2 \cos\theta = \widetilde{\Delta t}\, v_s \cos\theta}$$

$$\boxed{\cos\theta = \hat{v}_s \cdot \hat{n}}$$

$$\Delta t' = \frac{l_1}{c} = \frac{\Delta t\, v_s \cos\theta}{c}$$

additional time Interval that photon is traveling

$$= \frac{\Delta t_{rest}}{\sqrt{1 - v_s^2/c^2}} \frac{v_s \cos\theta}{c}$$

$$= \frac{\Delta t_{rest}}{\sqrt{1 - \left(\frac{v_s}{c}\right)^2}} \frac{\vec{v}_s \cdot \hat{n}}{c}$$

- Time dilation
- Displacement

$$\Delta t_{obs} = \frac{\Delta t_{rest}}{\sqrt{1 - \left(\frac{v_s}{c}\right)^2}} + \frac{\Delta t_{rest}}{\sqrt{1 - \left(\frac{v_s}{c}\right)^2}} \frac{\vec{v}_s \cdot \hat{n}}{c}$$

از دید ناظر ساکن.

$$\Delta t_{obs} = \frac{1}{f_{obs}}$$

$$\Delta t_{rest} = \frac{1}{f_s}$$

$$f_{obs} = f_s \left(1 - \frac{v_s^2}{c^2}\right)^{\frac{1}{2}} \left(1 + \frac{\vec{v}_s \cdot \hat{n}}{c}\right)^{-1}$$

$\downarrow$

$\lambda_{obs} = \ldots$

For $\theta = 0$ $\quad f_{obs} = f_s \sqrt{\dfrac{c - v_s}{c + v_s}}$

For $\theta = \pi/2$ $\quad f_{obs} = f_s \sqrt{1 - \dfrac{v_s^2}{c^2}} \neq f_s$

Classical Doppler Effect

For $\theta = \pi$ $\quad f_{obs} = f_s \sqrt{\dfrac{c + v_s}{c - v_s}}$

Redshift $\quad Z \equiv \dfrac{\lambda_{obs} - \lambda_s}{\lambda_s} = \dfrac{\Delta\lambda}{\lambda_s}$

$\boxed{\lambda f = c}$

$Z > 0 \longrightarrow$ Redshift

$Z < 0$ — Blueshift

Hubble Expansion : $\longrightarrow$ $v \sim R$

$\lambda_{Th} \sim \dfrac{1}{\sqrt{m K_B T \cdots 2\pi}}$

$\lambda = \dfrac{c}{p}$

$R = v t_H = \dfrac{v}{H}$

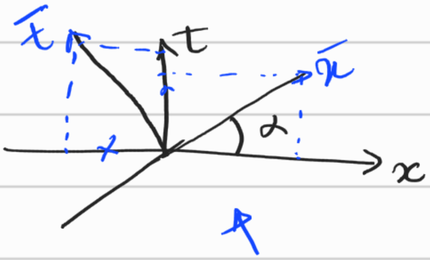
$R = a X$

$v = \dot{a} X \times \dfrac{a}{a} = \dfrac{\dot{a}}{a} \underbrace{X a}_{R}$

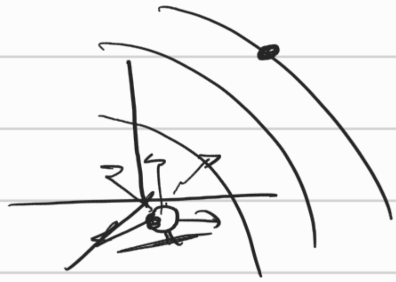
$\longrightarrow v = H R$

$\boxed{R_{obs-Un} \simeq c H^{-1}} \longleftarrow \boxed{R = \dfrac{v}{H}}$

15 Lorentz Transformation — as a rotation



$$* \quad \boxed{\tan \alpha = \frac{iu}{c}} \quad *$$



$x^2+y^2+z^2=c^2t^2 \longrightarrow$ light-like Event

$$\begin{cases} x^2+y^2+z^2-c^2t^2=0 \longrightarrow \text{scalar} \\ x'^2+y'^2+z'^2-c^2t'^2=0 \end{cases}$$

$$\begin{Bmatrix} x_1 \equiv x \\ x_2 \equiv y \\ x_3 \equiv z \\ x_4 \equiv ict \end{Bmatrix}$$

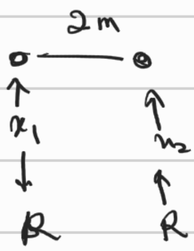
$$\underbrace{\sum_{i=1}^{4} (x_i - x_{i0})^2 = 0}_{S}$$

$$\underbrace{\sum_{i=1}^{4} (x'_i - x'_{i0})^2 = 0}_{\bar{S}}$$

همگنی فضا

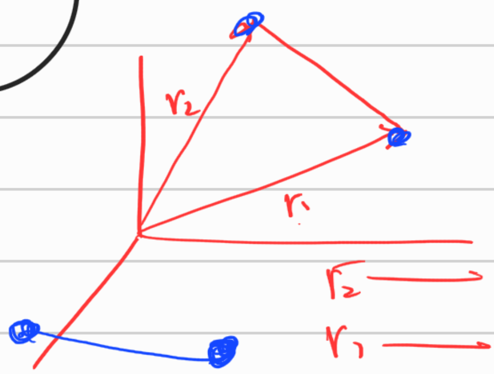
Homogeniety

$\Delta l = x_2 - x_1$



$\Delta l(x_2, x_1) = x_2 - x_1$

Δ



$|\bar{r}_1 - \bar{r}_2| \;=\; |(\bar{r}_1+\bar{R}) - (\bar{r}_2+\bar{R})|$

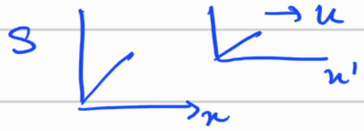
$r_2 \longrightarrow \vec{r}_2 + \vec{R}$

$r_1 \longrightarrow \vec{r}_1 + \vec{R}$

$$\left\{\begin{aligned} \bar{x}_1 &= x_1\cos\alpha + x_4\sin\alpha \\ \bar{x}_4 &= -x_1\sin\alpha + x_4\cos\alpha \\ \bar{x}_2 &= x_2 \\ \bar{x}_3 &= x_3 \end{aligned}\right\}$$

$$u = -ic\tan\alpha$$

$$\cos\alpha = \frac{1}{\sqrt{1-u^2/c^2}}$$



$$\left\{\begin{aligned} \bar{x} &= \frac{x-ut}{\sqrt{1-u^2/c^2}} & \bar{y} &= y \\ \bar{t} &= \frac{t-\frac{ux}{c^2}}{\sqrt{1-u^2/c^2}} & \bar{z} &= z \end{aligned}\right\}$$

$$\frac{u}{c} \to 0 \implies \left\{\begin{aligned} \bar{x} &= x-ut \\ \bar{y} &= y \\ \bar{z} &= z \\ \bar{t} &= t \end{aligned}\right\} \quad \text{Galilean Transformation}$$

(16) Four-Vector.

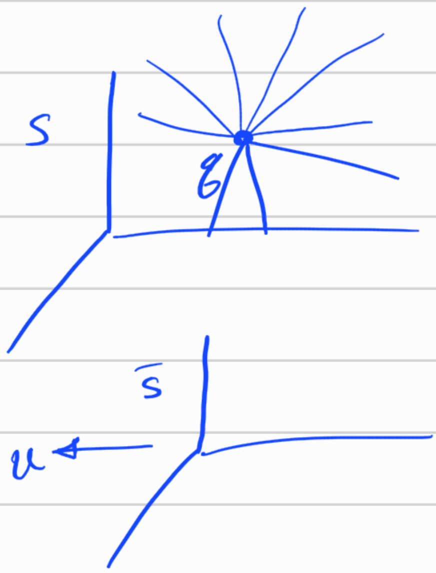
$$P = m_0(1-v^2/c^2)^{-1/2}(\vec{V}, ic)$$

$$F = \frac{d}{d\tau}(p, imc)$$

$$\boxed{\bar{V}\cdot\vec{F} = 0}$$

(17) Covariant Form of Maxwell Eq.

$$\left\{ \begin{array}{l} F_{ij} \equiv A_{j,i} - A_{i,j} \\ A = (A, i\phi) \\ \Box^2 A = -\frac{4\pi}{c} J \\ J = (j, ic\rho) \end{array} \right\}$$



$\vec{E} = \frac{q}{4\pi\epsilon_0} \frac{\hat{r}}{r^2}$

$\vec{B} = 0$