بسم الله الرحمن الرحیم

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Quintessence Model.

Boltzmann Equation in Expanding Universe

The cosmological constant problem and quintessence.

Varun Sahni
Inter-University Centre for Astronomy & Astrophysics, Puné 411 007, India

Abstract. I briefly review the cosmological constant problem and the issue of dark energy (or quintessence). Within the framework of quantum field theory, the vacuum expectation value of the energy momentum tensor formally diverges as k^4. A cutoff at the Planck or electroweak scale leads to a cosmological constant which is, respectively, 10^{123} or 10^{55} times larger than the observed value, $\Lambda/8\pi G \simeq 10^{-47}$ GeV^4. The absence of a fundamental symmetry which could set the value of Λ to either zero or a very small value leads to *the cosmological constant problem.* Most cosmological scenario's favour a large time-dependent Λ-term in the past (in order to generate inflation at $z \gg 10^{10}$), and a small Λ-term today, to account for the current acceleration of the universe at $z \lesssim 1$. Constraints arising from cosmological nucleosynthesis, CMB and structure formation constrain Λ to be sub-dominant during most of the intermediate epoch $10^{10} < z < 1$. This leads to the *cosmic coincidence* conundrum which suggests that the acceleration of the universe is a recent phenomenon and that we live during a special epoch when the density in Λ and in matter are almost equal. Time varying models of dark energy can, to a certain extent, ameliorate the fine tuning problem (faced by Λ), but do not resolve the puzzle of cosmic coincidence. I briefly review tracker models of dark energy, as well as more recent brane inspired ideas and the issue of horizons in an accelerating universe. Model independent methods which reconstruct the cosmic equation of state from supernova observations are also assessed. Finally, a new diagnostic of dark energy – 'Statefinder', is discussed.

1. Introduction

Einstein (1917) introduced the cosmological constant Λ because he believed that a closed static universe which emerged in the presence of both Λ and matter agreed

Quintessence Potential	Reference
$V_0 \exp(-\lambda\phi)$	Ratra & Peebles (1988), Wetterich (1988), Ferreira & Joyce (1998)
$m^2\phi^2, \lambda\phi^4$	Frieman et al (1995)
$V_0/\phi^\alpha, \alpha > 0$	Ratra & Peebles (1988)
$V_0 \exp(\lambda\phi^2)/\phi^\alpha, \alpha > 0$	Brax & Martin (1999,2000)
$V_0(\cosh\lambda\phi - 1)^p$,	Sahni & Wang (2000)
$V_0 \sinh^{-\alpha}(\lambda\phi)$,	Sahni & Starobinsky (2000), Ureña-López & Matos (2000)
$V_0(e^{\alpha\kappa\phi} + e^{\beta\kappa\phi})$	Barreiro, Copeland & Nunes (2000)
$V_0(\exp M_p/\phi - 1)$,	Zlatev, Wang & Steinhardt (1999)
$V_0[(\phi - B)^\alpha + A]e^{-\lambda\phi}$,	Albrecht & Skordis (2000)

Table 1.

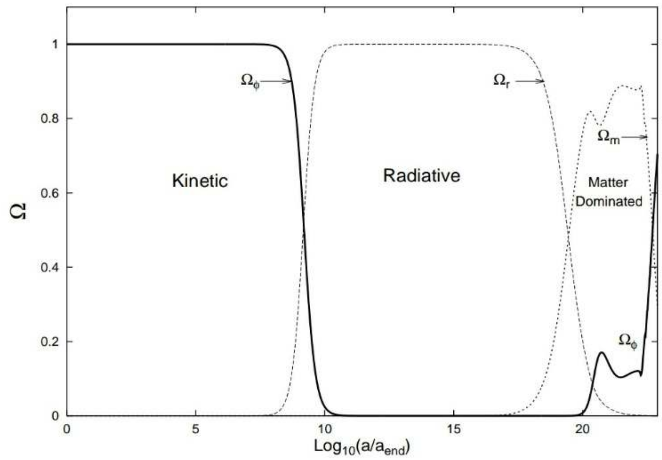

Figure 2. The post-inflationary density parameter Ω is plotted for the scalar field (solid line) radiation (dashed line) and cold dark matter (dotted line) in the quintessential-inflationary model decribed by (9) with $p = 0.2$. Late time oscillations of the scalar field ensure that the mean equation of state turns negative $\langle w_\phi \rangle \simeq -2/3$, giving rise to the current epoch of cosmic acceleration with $a(t) \propto t^2$ and present day values $\Omega_{0\phi} \simeq 0.7, \Omega_{0m} \simeq 0.3$. From Sahni, Sami and Souradeep [41].

Power-law Parameterized Quintessence Model

Sohrab Rahvar[1,2], M. Sadegh Movahed[1,2]
[1]*Department of Physics, Sharif University of Technology, P.O.Box 11365–9161, Tehran, Iran*
[2] *Institute for Studies in theoretical Physics and Mathematics, P.O.Box 19395-5531,Tehran, Iran*

We introduce a power-law parameterized quintessence model for the dark energy which accelerate universe at the low redshifts while behaves as an ordinary matter for the early universe. We construct a unique scalar potential for this parameterized quintessence model. As the observational test, the Supernova Type Ia (SNIa) Gold sample data, size of baryonic acoustic peak from Sloan Digital Sky Survey (SDSS), the position of the acoustic peak from the CMB observations and structure formation from the 2dFGRS survey are used to constrain the parameters of the quintessence model. The best fit parameters indicates that the equation of state of this model at the present time is less than one ($w_0 < -1$) which violates the energy condition in General Relativity. Finally we compare the age of old objects with age of universe in this model.
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I. INTRODUCTION

Observations of the apparent luminosity and redshift of type Ia supernovas (SNIa) provide the main evidence for the positive accelerating expansion of the Universe [1,2]. A combined analysis of SNIa and the Cosmic Microwave Background radiation (CMB) observations indicates that the dark energy filled about 2/3 of the total energy of the Universe and the remained part is dark matter with a few percent in the form of Baryonic matter from the Big Bang nucleo synthesis [3–5].

The "cosmological constant" is a possible solution for the acceleration of the universe [6]. This constant term in Einstein field equation can be regarded as an fluid with the equation of state of $w = -1$. However, there are two problems with the cosmological constant, namely the *fine-tuning* and the *cosmic coincidence*. In the framework

gas [14] provide various equation of states for the dark energy [13,15–21].

There are also phenomenological models, parameterize the equation of state of dark energy in terms of redshift [22–24]. For a dark energy with the equation of state of $p_X = w_X\rho_X$, using the continuity equation, the density of dark energy changes with the scale factor as:

$$\rho_X = \rho_X^{(0)} a^{-3(1+\bar{w}_X(a))}, \qquad (1)$$

where $\bar{w}_X(a)$ is the mean of the equation of state in the logarithmic scale:

$$\bar{w}_X(a) = \frac{\int w_X(a) d\ln(a)}{\int d\ln(a)} \qquad (2)$$

The main aim of these models is to cure the fine-tuning problem of the dark energy density by means that the

Part 1

① Field theoretic approach to construct Quintessence Model.

$$S = S_{EH} + S_{\phi}$$

$\leftarrow$ Quintessence Field $\rightarrow \rho_{\phi}$

Dark Energy

$$-1 \leqslant \omega_{\phi} \lessapprox -1/3$$

$$② \quad S = \int d^4x \sqrt{-g} \left[\frac{M_{PL}^2}{2} R + \underbrace{\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)}_{S_\phi} \right]$$

$\delta S = 0 \longrightarrow$ Euler-Lagrange

$$③ \begin{cases} \rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \end{cases} \Longrightarrow \omega_\phi = \frac{P_\phi}{\rho_\phi}$$

$$\boxed{\omega_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}}$$

How to reconstruct the $V(\phi)$ and $\phi(t)/\phi(z)$

for a given $\omega_\phi = \checkmark = \omega_\phi(z; \{\ell\})$

Non-Intracting.

④ $$\dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = 0$$

$$\rho_\phi(z) = \rho_\phi^\circ \, \underbrace{e^{3\int_0^z (1+\omega_\phi(z,\{\ell\}))\, d\ln(1+z)}}$$

$$\rho_\phi(t) \simeq \rho_\phi(z) \equiv \rho_\phi^\circ \, U(z; \{\ell\})$$

Parameters for model.

⑤ from ③ $\omega_\phi = \frac{P_\phi}{\rho_\phi}$ $\Rightarrow$

$$\begin{cases} \frac{1}{2}\dot{\phi}^2 = \frac{1}{2}(1+\omega_\phi(z,\{\ell\}))\,\rho_\phi \\ V(\phi) = \frac{1}{2}(1-\omega_\phi(z,\{\ell\}))\,\rho_\phi \end{cases}$$

⑥ $$\boxed{V(\phi) = \frac{1}{2}(1-\omega_\phi(z;\{\ell\}))\,\rho_\phi^\circ \, U(z,\{\ell\}) \quad \text{☆ } V(z)}$$

Rest free Parameters

⑦ $$H^2(z,\{\ell\},\{\theta\}) = H_0^2\left[\Omega_m^\circ(1+z)^3 + \Omega_r^\circ(1+z)^4 + \Omega_\phi^\circ\, U(z,\{\ell\})\right]$$

Quintessence Component

Flat Univ $K=0$

$$\Omega_\phi^\circ \equiv \frac{\rho_\phi^\circ}{3M_{PL}^2 H_0^2}$$

$$\{\theta\} : \{\Omega_m^\circ, \Omega_r^\circ, H_0\}$$

⑧ $$\frac{d\phi}{dt} = \pm \left(1 + w_\phi(z; \{\theta\})\right)^{1/2} \rho_\phi^{1/2}$$

$\dot{\phi}_0 > 0 \rightarrow +$

$\dot{\phi}_0 < 0 \rightarrow -$

$$\frac{d\phi}{dz} \frac{dz}{da} \frac{da}{dt} = \;\; //$$

☆ $$\boxed{\frac{d\phi}{dz} = \pm \frac{a(1+w_\phi)^{1/2} \rho_\phi^{1/2}}{H} = \pm \frac{(1+w_\phi)^{1/2} \rho_\phi^{1/2}}{H(1+z)}}$$

(+)

$$\frac{d\phi}{dz} = + \sqrt{3}\, M_{PL} \frac{(1+w_\phi)^{1/2}}{1+z} \left[1 + \frac{\Omega_m^\circ}{\Omega_\phi^\circ} (1+z)^3 \times \frac{1}{U(z, \{\theta\})} \right]^{-1/2}$$

$$\tilde{\phi} \equiv \frac{\phi}{M_{PL}} , \quad \tilde{V} \equiv \frac{V}{\rho_\phi}$$

or

☆ $$\frac{d\tilde{\phi}}{dz} = \sqrt{3} \frac{(1+w_\phi)^{1/2}}{(1+z)} \left[1 + \frac{\Omega_m^\circ}{\Omega_\phi^\circ} (1+z)^3 / U(z, \{\theta\}) \right]^{-1/2}$$

☆ $\tilde{V}[\tilde{\phi}] = \frac{1}{2}\left(1 - \omega_\phi(z, \{\ell\})\right) U(z, \{\ell\})$

Ex: $w(a) = w_0 a^\alpha (1 + \ln a^\alpha) \equiv \omega_\phi(z, \{\ell\}) = \omega_\phi(z, \{\alpha, \omega_0\})$

CPL. $\omega(a) = \omega_0 + \omega_1(1-a)$ for $a \sim 1$

for $z \sim 0$

For $\omega_0 = -1$

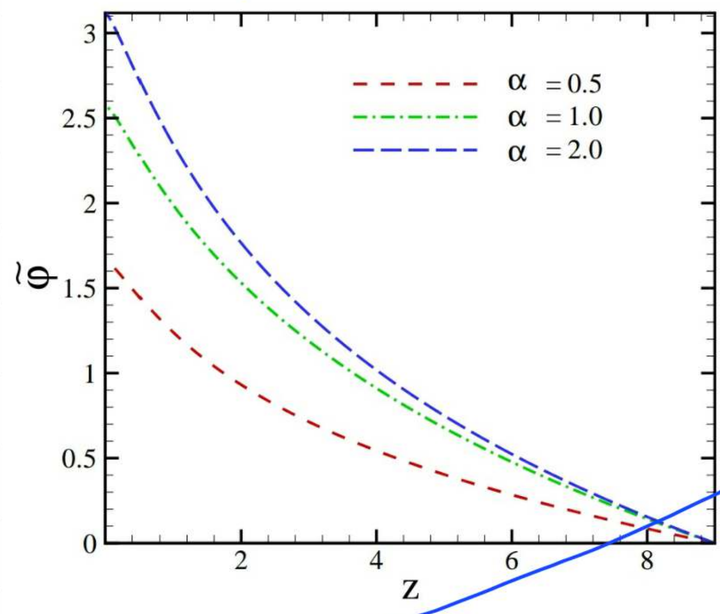

FIG. 3. Dependence of scalar field in terms of redshift for the power-law Quintessence model.

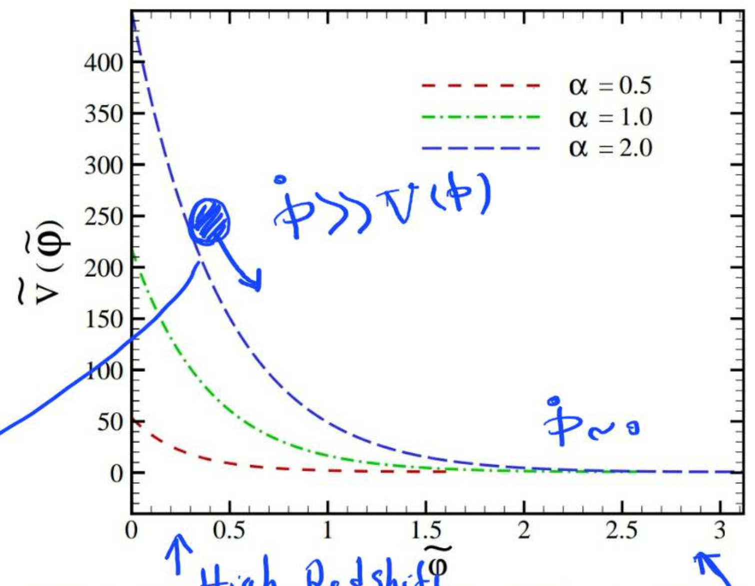

FIG. 4. Reconstruction of the scalar field potential as a function of scalar field for the power-law Quintessence model.

Low Redshift

Physical Explanation of mentioned Results:

at small z, we must have $\omega_\phi < -1/3$ to have accelerated Expansion. To this end the $|\dot{\phi}|^2 \ll V(\phi)$ and For High redshift according to Figure; $|\dot{\phi}|^2 \gg V(\phi)$ and $\omega_\phi > 0$ and at low Redshift $|\dot{\phi}| \sim 0$ and $\omega_\phi \sim -1$

Part 2:

Section 3.2 Boltzmann Equation

① According to statistical sense, we try to examine the probability of finding particles in given state in phase space:

$$\frac{d^{3N}q \, d^{3N}p}{h^{3N}}$$ element of phase space

$$\dot{q}_i = +\frac{\partial H}{\partial p_i}$$

$$\dot{p}_i = -\frac{\partial H}{\partial q_i}$$

Probability of finding particles at $\{(q,p)\}$ and $\{(q+dq),(p+dp)\}$ is

② $$\left\{ \underbrace{f(\{q,p\})}\, \frac{d^{3N}q \, d^{3N}p}{h^{3N}} \right\}$$

$$\frac{N}{V} = \left\langle \sum_{i=1}^{N} \delta_D(\vec{q} - \vec{q}_i) \right\rangle = \int \frac{d^{3N}q \, d^{3N}p}{h^{3N}} f(\{q,p\}) \sum \delta_D(\vec{q}-\vec{q}_i)$$

Ex: Ideal Gas $Z = \left(\frac{V}{\lambda^3}\right)^N$

$$\rho = \frac{e^{-\beta H}}{Z}$$

$$\left\langle \sum \delta_D(\bar{q}-\bar{q}_i) \right\rangle = \int \frac{d^{3N}q \, d^{3N}p}{h^{3N}} \sum \delta_D(\bar{q}-\bar{q}_i) \times \rho$$

$$= \frac{1}{V} \sum_{i=1}^{N} \int d^3 q_i \, \delta_D(\bar{q}-\bar{q}_i) = \frac{N}{V}$$

(3) $\{\bar{q}, \bar{p}\} \longrightarrow \{\bar{x}, \bar{p}\}$ Dodelson Notation

$$N = \sum_k \langle n_k \rangle = \int \frac{d^3k}{\left(\frac{2\pi}{L}\right)^3} \langle n(k) \rangle$$

$$N(\bar{x}, \bar{p}; t) = f(\bar{x}, \bar{p}, t) \frac{d^3x \, d^3p}{(2\pi)^3}$$

$$N = \int d\epsilon \, g(\epsilon) \langle n(\epsilon) \rangle = \int d\epsilon \, g(\epsilon) f(\epsilon, t)$$

Recall that in Cosmology 14 – 15 lecture

$$dn_i = g_i \, d^3p \, f(p, T)$$

$$dN_i = g_i \, d^3p \, d^3x \, f$$

Therefore. the Number of Particles

in a Small Portion of

phase space is

$$\boxed{N(\bar{x}, \bar{p}; t) = f(\bar{x}, \bar{p}; t) \frac{d^3x \, d^3p}{(2\pi)^3}}$$

④ $\dfrac{df}{dt} = ?$ Boltzmann Equat indicates this evolution

$$\frac{df}{dt} = \left.\frac{\partial f}{\partial t}\right|_{\text{external Force}} + \left.\frac{\partial f}{\partial t}\right|_{\text{diffusion}} + \left.\frac{\partial f}{\partial t}\right|_{\text{Collision}}$$

We ignore collision term and we consider

No sink No source

$$\underbrace{N(\bar{x},\bar{p},t)} = \underbrace{N(\bar{x}+d\bar{x},\bar{p}+d\bar{p},t+dt)}$$

$$f(\bar{x},\bar{p},t)\,\underbrace{\frac{d^3x\,d^3p}{(2\pi)^3}}_{d\Omega(x,p,t)} = f(\bar{x}+d\bar{x},\bar{p}+d\bar{p};t+dt) \times \underbrace{\frac{d^3x\Big|_{t+\Delta t}\,d^3p\Big|_{t+\Delta t}}{(2\pi)^3}}_{d\Omega(x+dx,p+dp,t+dt)}$$

$$d\Omega\Big|_t = d\Omega_{t+dt}$$ ↳ Liouville's theorem

$$\boxed{\frac{df}{dt}=0} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial \bar{x}}\cdot\frac{d\bar{x}}{dt} + \frac{\partial f}{\partial \bar{p}}\cdot\frac{d\bar{p}}{dt}$$

With collision term

$$\frac{df}{dt} = C[f]$$

$$\bar{V}\cdot\nabla f \qquad \bar{F}\cdot\frac{\partial f}{\partial p}$$

$$C[f] = \frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial \bar{x}}\cdot\frac{d\bar{x}}{dt} + \frac{\partial f}{\partial \bar{p}}\cdot\frac{dp}{dt}$$

Recall that

Continuity Eq $\boxed{\frac{\partial \rho}{\partial t} + \underbrace{\bar{\nabla}\cdot(\rho\bar{v})}_{} = 0}$

this is related to compersibility of fluid

$$\bar{\nabla}\rho\cdot\bar{v} + \rho\,\underbrace{\bar{\nabla}\cdot\bar{v}}_{}$$

and for Boltzmann Equad we consider incompersible Point of phase space

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Boltzmann _ Lisuville's operator:

$$C[f] = 0 \implies \frac{df}{dt} = 0$$

$$\frac{\partial f}{\partial t} = -\dot{\vec{x}}\cdot\bar{\nabla}_x f - \dot{\vec{P}}\cdot\bar{\nabla}_p f$$

$$\boxed{\frac{\partial f}{\partial t} = -\frac{\bar{P}}{m}\cdot\bar{\nabla}_r f - m\vec{a}\cdot\nabla_p f}$$