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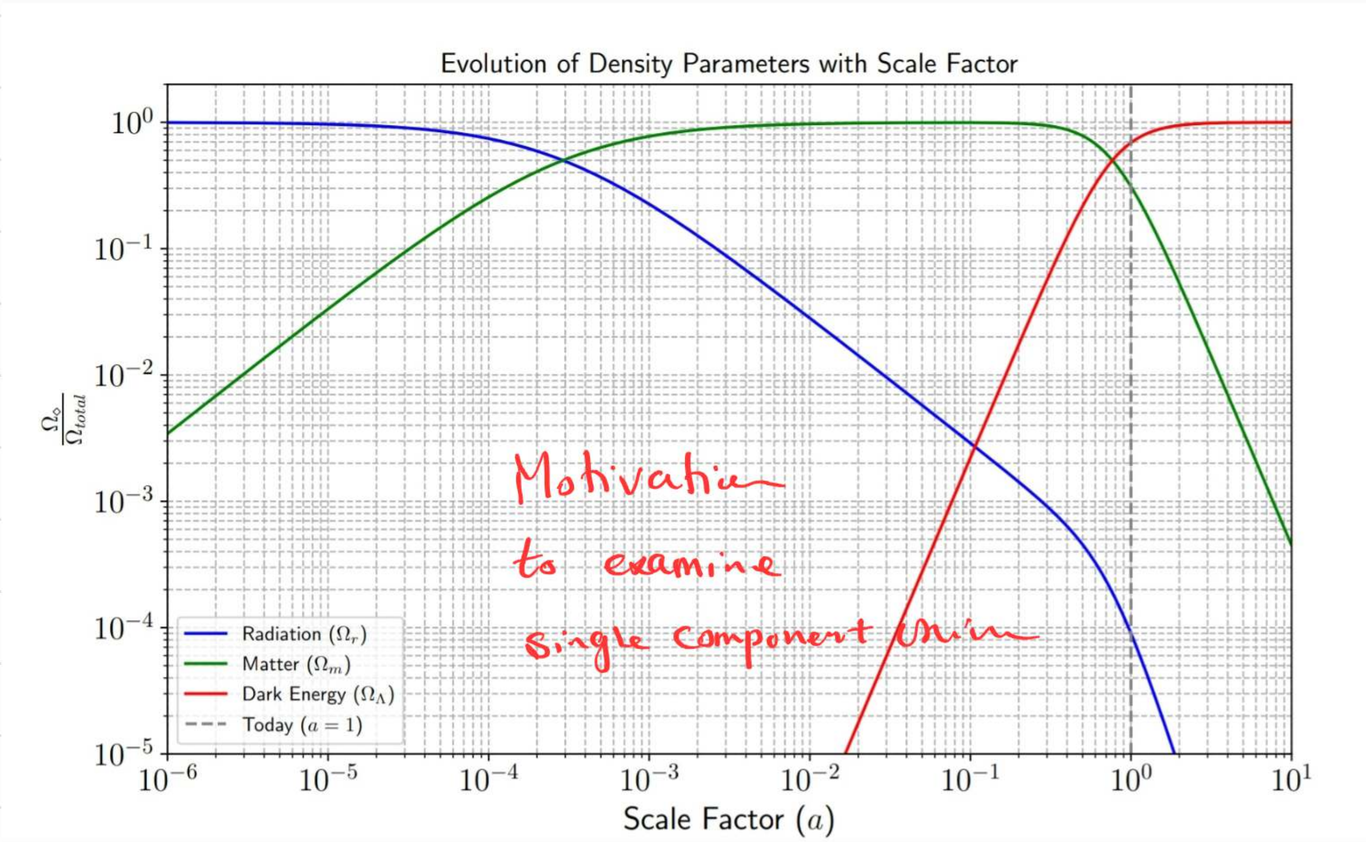
Friedmann Equation : Continuation

{Single Component Universe}



☆ $$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{total} - \frac{K}{a^2} \qquad \rho_{total} = \sum_{i=1} \rho_i$$

For ΛCDM model $\quad \rho_{total} = \rho_r + \rho_m + \rho_\Lambda$

☆ $$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho_{total} + 3P_{total}\right) \qquad P_{total} = \sum_i P_i$$

☆ $$\frac{d\rho_{tm}}{dt} + 3\frac{\dot{a}}{a}(\rho_{t} + P_{t}) = 0$$

For non-Intracting models we achieve N-disjoint Equ

☆ $$\boxed{P_i = \omega_i \rho_i}$$ $\dot{\ } \equiv \frac{d}{dt}$, $' \equiv \frac{d}{d\eta}$ ← Conformal Time

$$\boxed{d\eta = \frac{dt}{a}}$$

① Radiation Dominated era : $\boxed{\omega_r = 1/3 = cts}$

$$\dot{\rho}_r + 3\frac{\dot{a}}{a}(\rho_r + P_r) = 0 \Rightarrow \rho_r(a) = \rho_r(a = a_0 = 1)\, a^{-4}$$

↓

Recall that $\rho_r = \frac{h\nu}{V} \sim \frac{a^{-1}}{a^3} \sim a^{-4}$

$\underline{\underline{H}}$, $\underline{\underline{a(t)}}$, $\underline{\underline{t}}$, open / Flat / closed (spatial curvature)

$d\eta = \frac{dt}{a(t)}$ { Recall that $ds^2 = -dt^2 + a^2(t)[d\chi^2 + \chi^2 d\Omega^2]$

$ds^2 = a^2(t)[-d\eta^2 + d\chi^2 + \chi^2 d\Omega^2]$ }

Conformal Time.

$$\frac{d}{dt} = \frac{d}{d\eta}\frac{d\eta}{dt} = a^{-1}\frac{d}{dt} \quad , \quad \frac{\dot{a}}{a} = \frac{a'}{a^2} \equiv \frac{h}{a}$$

$$h \equiv \frac{a'}{a} \quad , \quad H \equiv \frac{\dot{a}}{a}$$

$$\frac{h^2}{a^2} = H^2 = \frac{8\pi G}{3} \rho_r^0 a^{-4} - K/a^2$$

$$h^2 + K = \frac{8\pi G}{3} \rho_r^0 a^{-2} \qquad \text{First Eq.}$$

$$h^2 + 2h' + K = -\frac{8\pi G}{3} \rho_r^0 a^{-3} \qquad \text{Second Eq.}$$

☆ $$h(\eta) = \begin{cases} \mathrm{Cotg}(\eta) & K>0 \quad \text{closed} \\ \frac{1}{\eta} & K=0 \quad \text{Flat} \\ \mathrm{Coth}(\eta) & K<0 \quad \text{open} \end{cases}$$

$$\Omega_K^0 = 1 - \Omega_{tot}^0 \equiv -K/H_0^2$$

☆ $$a(\eta) = \begin{cases} \sin(\eta) & K>0 \\ \eta & K=0 \\ \sinh(\eta) & K<0 \end{cases}$$

☆ $$t(\eta) = \begin{cases} 1-\cos(\eta) & K>0 \\ \eta^2/4 & K=0 \\ \cosh(\eta) & K<0 \end{cases}$$

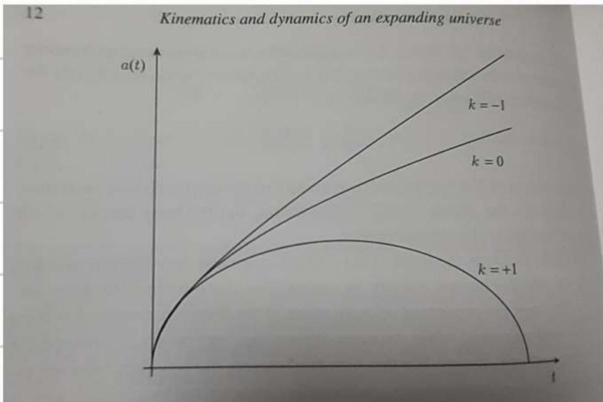


Fig. 1.3.

For $K=0$: $t(\eta) \sim \eta^2$, $\eta \sim t^{1/2}$; $a(\eta) \sim \eta$

$$t \sim a^2 \rightarrow \boxed{a \sim t^{1/2}}$$

$$\boxed{H = \frac{1}{2t}}$$

$$\boxed{T \sim a^{-1}}$$

{ Recall

$$dU = \delta Q + \delta W$$
$$= TdS - PdV$$
$$dU = 0 - PdV$$
$$d(\alpha T^4 V) = -PdV$$
$$\alpha T^4 dV + 4\alpha V T^3 dT = -PdV$$

$$P = \tfrac{1}{3}\frac{U}{V}$$

$$4U\frac{dT}{T} = -\tfrac{1}{3}U\frac{dV}{V} - U\frac{dV}{V}$$

$$4U\frac{dT}{T} = -\tfrac{4}{3}U\frac{dV}{V} \rightarrow \boxed{T \sim a^{-1}}$$

② Cold Dark matter dominated era. $\boxed{\omega_m = 0} \Rightarrow P_m = 0$

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}(\rho_m + P_m) = 0 \Rightarrow \boxed{\rho_m(a) = \rho_m^0 a^{-3}}$$

[Recall that $P = \omega(a)\rho \Rightarrow \rho \sim a^{-3(1+\bar{\omega}(a))}$

$$\bar{\omega}(a) \equiv \frac{\int d\ln a\, \omega(a)}{\int d\ln a}$$]

☆ $$h(\eta) = \begin{cases} \operatorname{Cotg}(\eta/2) & K > 0 \\ 2/\eta & K = 0 \\ \coth(\eta/2) & K < 0 \end{cases}$$

☆ $$a(\eta) = \begin{cases} 1 - \cos(\eta) & K > 0 \\ \eta^2/2 & K = 0 \\ \cosh(\eta) - 1 & K < 0 \end{cases}$$

☆ $$t(\eta) = \begin{cases} \eta - \sin(\eta) & K > 0 \\ \eta^3/6 & K = 0 \\ \sinh(\eta) - \eta & K < 0 \end{cases}$$

For Flat Universe $K = 0$ $\quad a \sim \eta^2, \quad t \sim \eta^3, \quad \eta \sim t^{1/3}$

$$t \sim a^{3/2}$$

$$\boxed{a \sim t^{2/3}} \rightarrow \text{For dust}$$

CDM

$$\boxed{H = \frac{2}{3t}}$$

③ Λ dominated era : $w_\Lambda = -1 \Rightarrow \rho_\Lambda = cts$

$$H^2 = \frac{8\pi G}{3}\rho_\Lambda = cts \qquad \dot{a} \propto a$$

$$a(t) \sim e^{\sqrt{\frac{8\pi G}{3}\rho_\Lambda}\, t} \longrightarrow H = cts$$

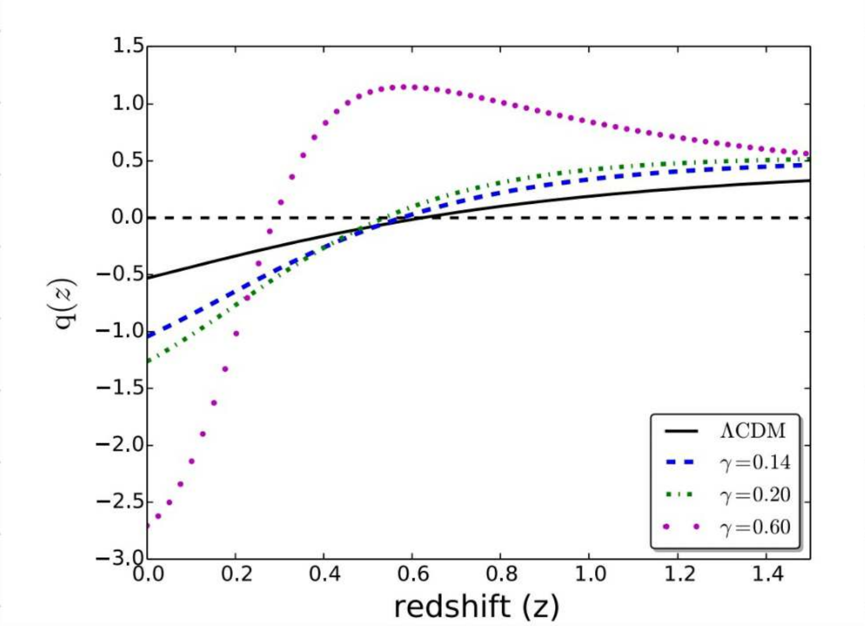
$$a(t) = \begin{cases} \cosh\left(\sqrt{\frac{8\pi G}{3}\rho_\Lambda}\, t\right) & K > 0 \\ e^{\sqrt{\frac{8\pi G}{3}\rho_\Lambda}\, t} & K = 0 \\ \sinh\left(\sqrt{\frac{8\pi G}{3}\rho_\Lambda}\, t\right) & K < 0 \end{cases}$$

For $K = 0$ $\eta = cts$

Era	Curvature (k)	Scale Factor $a(t)$	Hubble Parameter $H(t)$	Conformal Time $\eta(t)$
Radiation	Flat ($k = 0$)	$a \propto t^{1/2}$	$H = \frac{1}{2t}$	$\eta \propto t^{1/2}$
	Closed ($k = +1$)	$a = a_{\max} \sin\eta$	-	Parametric solution
	Open ($k = -1$)	$a = a_{\min} \sinh\eta$	-	Parametric solution
Matter	Flat ($k = 0$)	$a \propto t^{2/3}$	$H = \frac{2}{3t}$	$\eta \propto t^{1/3}$
	Closed ($k = +1$)	$a = \frac{B}{2}(1 - \cos\eta)$	-	Cycloidal solution
	Open ($k = -1$)	$a = \frac{B}{2}(\cosh\eta - 1)$	-	Hyperbolic solution
Dark Energy (Λ)	Flat ($k = 0$)	$a \propto e^{Ht}$	$H = \text{constant}$	$\eta \to \text{constant}$
	Closed ($k = +1$)	$a = \frac{1}{H}\cosh(Ht)$	-	Finite conformal horiz
	Open ($k = -1$)	$a = \frac{1}{H}\sinh(Ht)$	-	Infinite conformal tim

(4) Cosmographic Parameters

Deceleration Parameter $q \equiv -\frac{\ddot{a}}{aH^2}$



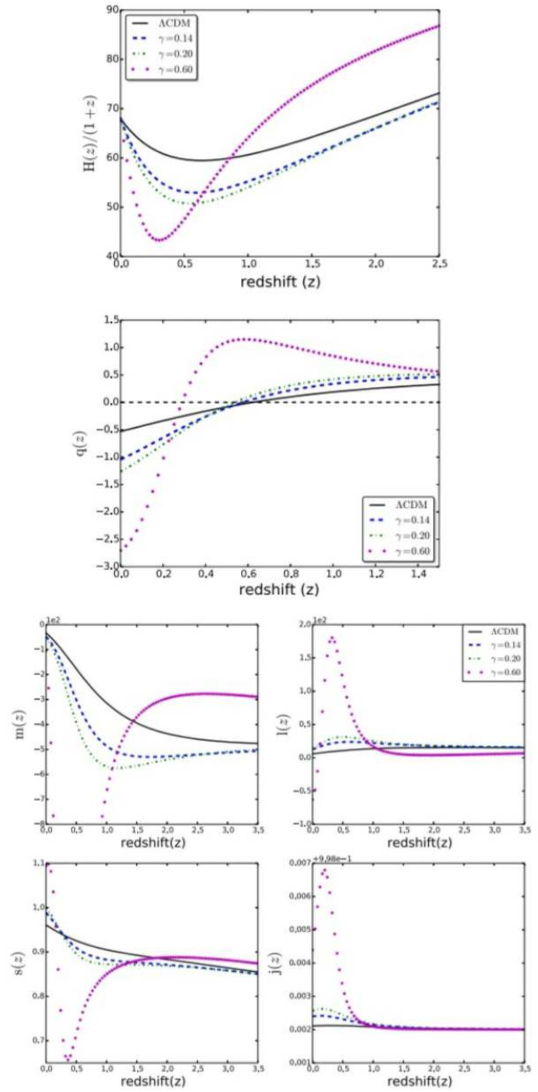
other Cosmographic Parameters are as follows:

$$j \equiv \frac{1}{a\,H^3}\frac{\mathrm{d}^3 a}{\mathrm{d}t^3}, \quad s \equiv \frac{1}{a\,H^4}\frac{\mathrm{d}^4 a}{\mathrm{d}t^4},$$
$$l \equiv \frac{1}{a\,H^5}\frac{\mathrm{d}^5 a}{\mathrm{d}t^5}, \quad m \equiv \frac{1}{a\,H^6}\frac{\mathrm{d}^6 a}{\mathrm{d}t^6}. \tag{36}$$

these parameters are called *jerk*, *snap*, *lerk* and *maxout*, respectively. It is worth noting that these parameters are not

$$\begin{aligned}
\dot{H} &= -H^2(1+q),\\
\ddot{H} &= H^3(j+3q+2),\\
\dddot{H} &= H^4[s-4j-3q(q+4)-6],\\
\ddddot{H} &= H^5[l-5s+10(q+2)j+30(q+2)q+24],\\
H^{(5)} &= H^6\{m-10j^2-120j(q+1)\\
&\quad -3[2l+5(24q+18q^2+2q^3-2s-qs+8)]\}.
\end{aligned} \tag{37}$$

Fig. 9 *Upper panel* the $\frac{H(z)}{(1+z)}$ as a function of the redshift. *Middle panel* deceleration probe diagnostic. *Lower panel jerk, snap, lerk* and *maxout* parameters for the bulk viscous model with respect to ΛCDM model. *Solid lines* represents corresponding quantity for ΛCDM. *Other lines* are associated with different values for viscosity. The rest of free parameters have been fixed according to JLA observation at 1σ confidence level



$$a(t) = a(t_0) + (t-t_0)\,\dot a(t_0) + \frac{(t-t_0)^2}{2!}\,\ddot a(t_0) + \dots$$

$$H(t) = H(t_0) + (t-t_0)\,\dot H(t_0) + \frac{(t-t_0)^2}{2!}\,\ddot H(t_0) + \dots$$

$$z = a^{-1} - 1$$

$$a^{-1} = a^{-1}(t_0) - (t-t_0)\,H_0 + \tfrac{1}{2} H_0^2 (t-t_0)^2 \left[2 + q_0\right] + \dots$$

$$q_0 \equiv -\frac{\ddot a}{aH}\bigg|_{t=t_0}$$

(5) Om diagnostic

4.6 *Om* diagnostic

The Om diagnostic method is indeed a geometrical diagnostic which combines Hubble parameter and redshift. It can differentiate dark energy model from ΛCDM. Sahni and his collaborators demonstrated that, irrespective to matter density content of Universe, acceleration probe can discriminate various dark energy models [67]. $Om(z)$ diagnostic for our spatially flat Universe reads

$$Om(z; \{\Theta_P\}) \equiv \frac{\mathscr{H}^2(z; \{\Theta_P\}) - 1}{(1+z)^3 - 1}. \tag{38}$$

where $\mathscr{H} \equiv \frac{H}{H_0}$ and H is given by Eq. (14). For ΛCDM model $Om(z) = \Omega_m^0$ while for other dark energy models, $Om(z)$ depends on redshift [68]. Phantom like dark energy corresponds to the positive slope of $Om(z)$ whereas the negative slope means dark energy behaves like quintessence [70].

$\omega < -1$

$\omega < 0$

$-1 < \omega < 0$

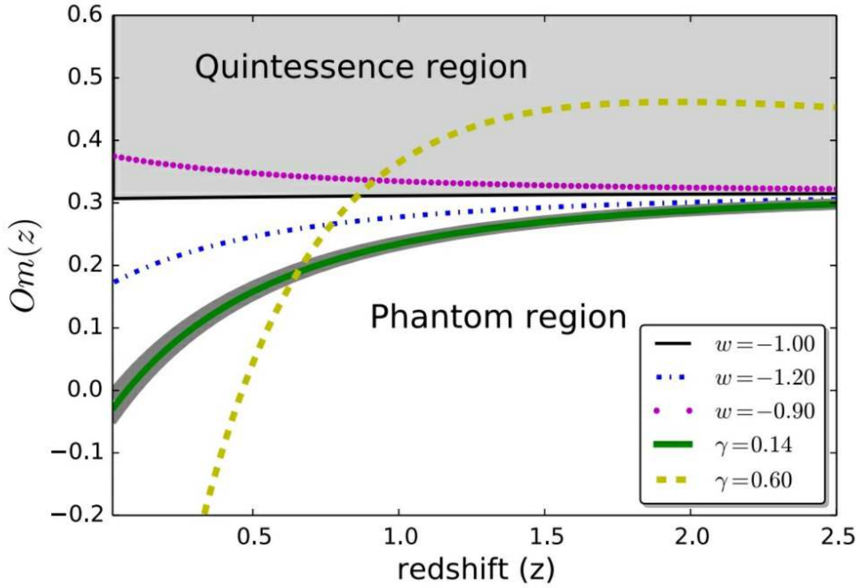


Fig. 10 $Om(z)$ diagnostic: *solid lines* represents $Om(z)$ for the cosmological constant. *Dashed* and *dot-dashed lines* represents $Om(z)$ for ΛCDM with $w = -0.90$ and $w = -1.20$, respectively. *Thick solid line* with corresponding 1σ confidence interval determined by JLA observation represents $Om(z)$ for the viscous dark energy model for $\gamma = 0.14$. A *long-dashed line* corresponds to $\gamma = 0.60$ demonstrating that dynamical dark energy model has almost quintessence behavior during the evolution of the Universe

6

4.7 Sandage–Loeb test

Another interesting measure is Sandage–Loeb test [72]. This criterion assesses the redshift drift of the Lyman-α spectra forest observed for distant quasars in the range of $2 \leq z \leq 5$ [73–75]. This quantity is defined by

$$\Delta v \equiv \frac{c\Delta z}{1+z} = cH_0\Delta t_0\left(1 - \frac{\mathscr{H}(z)}{1+z}\right), \tag{39}$$

where c is the speed of light. Expansion history of our cosmos can be examined by Sandage–Loeb test corresponding to its direct geometric measurement. Δt_0 is observation time interval in Eq. (39). Figure 11 indicates dimensionless quantity, $\Delta v/cH_0\Delta t_0$, versus redshift. Higher value of the viscosity leads to more deviation from ΛCDM model.

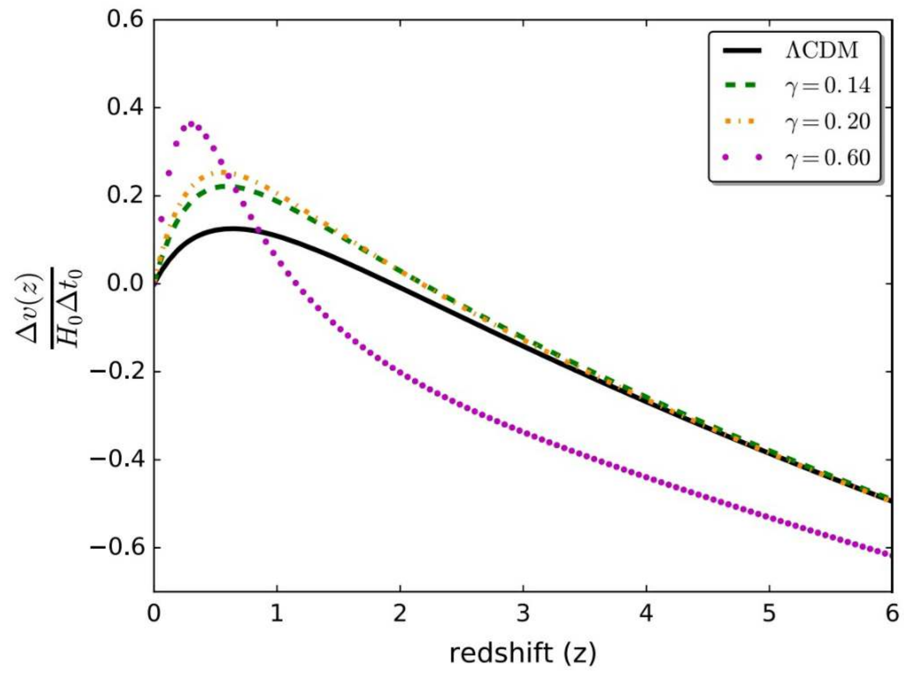


Fig. 11 The dimensionless redshift drift for various values of the viscosity of the viscous dark energy model. At the early time, the value of this parameter is more than the standard model due to contribution of the viscosity. In this plot we assume that $c = 1$

7 Distance duality

$$D_L = (1+z)\int \frac{dz}{H(z)}, \quad D_A = \frac{1}{(1+z)}\int \frac{dz}{H(z)}$$

$\frac{D_L}{D_A} = (1+z)^2$ If $GR + \Lambda$

Implications of distance duality violation for the H_0 tension and evolving dark energy

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We investigate whether a violation of the distance duality relation (DDR), $D_L(z) = (1+z)^2 D_A(z)$, connecting the angular diameter and luminosity distances, can explain the Hubble tension and alter the evidence for dynamical dark energy in recent cosmological observations. We constrain five phenomenological parameterisations of DDR violation using Baryon Acoustic Oscillation measurements from the DESI survey calibrated with the sound horizon derived from *Planck* Cosmic Microwave Background data and the Pantheon+ Type Ia supernova (SNIa) catalogue calibrated with the supernova absolute magnitude from SH_0ES. We find that two toy models can resolve the tension: a constant offset in the DDR (equivalent to a shift in the calibration of the SNIa data), $D_L(z)/D_A(z) \simeq 0.925(1+z)^2$, which leaves the hint for evolving dark energy unaffected; or a change in the power-law redshift-dependence of the DDR, restricted to $z \lesssim 1$, $D_L(z)/D_A(z) \simeq (1+z)^{1.866}$, together with a *constant* phantom dark energy equation of state $w \sim -1.155$. The Bayesian evidence slightly favours the latter model. Our phenomenological approach motivates the investigation of physical models of DDR violation as a novel way to explain the Hubble tension.

I. INTRODUCTION

The canonical Λ-cold-dark-matter (ΛCDM) model, in which dark energy is described by a cosmological constant Λ and dark matter behaves as a dust-like component, has been remarkably successful in explaining a wide range of high-precision cosmological observations, including (but not limited to) the cosmic microwave background (CMB) radiation [1–4], baryon acoustic oscillations (BAO) [5, 6], and the apparent magnitude of type Ia supernovae (SNIa) [7].

However, over the past decade, anomalies have emerged which raise questions about the validity of the ΛCDM model. There are indications that additional physical ingredients may be needed to restore concordance between datasets [8]. The most significant challenge to ΛCDM is the 'Hubble tension', a $\sim 5\sigma$ discrepancy between the direct measurement of the Hubble parameter H_0 – which measures the expansion of the Universe today – using Cepheid-calibrated SNIa from the Supernovae H_0 for the Equation of State (SH_0ES) programme [9] and the ΛCDM prediction calibrated with CMB data measured by the *Planck* satellite [1]. Systematic errors in H_0 estimates are actively being searched for, *e.g.* [9–22], and solutions to the tension which propose new physics have also been suggested [8, 23–25].

This tension can be traced to a mismatch in the *calibration* of the cosmological distance indicators: the SNIa and the BAO observed in various tracers of the density field. When both the SH_0ES absolute magnitude calibration and the *Planck* ΛCDM sound horizon calibration are applied, a substantial discrepancy arises between the SNIa and BAO data [26–30]. Under the ΛCDM model, this tension results in values for H_0 derived from the two datasets that are incompatible at 5σ. This incompatibility is also reflected in other parameters, in particular the physical matter density $\Omega_m h^2$ (where Ω_m is the fractional density of baryons and cold dark matter in the Universe and $h \equiv H_0/(100\,\mathrm{km\,s^{-1}\,Mpc^{-1}})$) [30–33], and is known as the 'cosmic calibration tension'. Moreover, even in the absence of the SH_0ES calibration, hints of deviations from ΛCDM have appeared when combining BAO and SNIa data, with a preference for dynamical dark energy at the $3-4\sigma$ level depending on the exact datasets used [34–37].

From a more fundamental perspective, these tensions can be seen as a tension between two sets of cosmic distance measurements. Cosmological observations infer either the *angular diameter distance*, $D_A(z)$, from the apparent angular size of an object of known physical size, or the *luminosity distance*, $D_L(z)$, from the flux of a source of known intrinsic brightness. The cosmic calibration tension arises when comparing luminosity and angular diameter distances. However, this comparison is made with the (often implicit) assumption that the distance-duality relation (DDR) [38, 39], $D_L(z) = (1+z)^2 D_A(z)$, holds in our Universe, allowing for direct conversion between $D_A(z)$ and $D_L(z)$.

The DDR holds in any cosmological model that satisfies the following conditions: i) spacetime is described by a pseudo-Riemannian manifold, ii) photons propagate along (unique) null geodesics, and iii) their number is conserved over time. DDR violations can range from the prosaic, such as photon scattering off dust or free electrons, to the exotic [40, 41]. They can emerge in modified theories of gravity where spacetime is not described by a pseudo-Riemannian manifold [42, 43], such as in torsion-based theories, where the connection is not purely Levi-
nergy–
ons in-

II. THE DDR, DISTANCE MEASUREMENTS AND COSMOLOGY

The DDR states that [38, 39]

$$D_L(z) = (1+z)^2 D_A(z)\,, \tag{1}$$

where $D_L(z)$ is the *luminosity distance* and $D_A(z)$ is the *angular diameter distance*. Our analysis assumes that the cosmological principle holds – *i.e.*, that the Universe is homogeneous and isotropic on sufficiently large scales. Consequently, we do not consider any directional dependence of the DDR but instead focus on its possible redshift dependence.[2] Under these assumptions, measurements of luminosity and angular diameter distances at different redshifts can be used to test the DDR. These tests can be performed either by considering specific physical models that predict DDR violations (hence constraining the models using data) or in an empirical model-agnostic way by rewriting the DDR as

$$\eta(z) = \frac{D_L(z)}{(1+z)^2 D_A(z)}\,, \tag{2}$$

where deviations from $\eta(z) = 1$ indicate a violation of the DDR.

⑧ Recent challenge about Cosmological Constant

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Extended Dark Energy analysis using DESI DR2 BAO measurements

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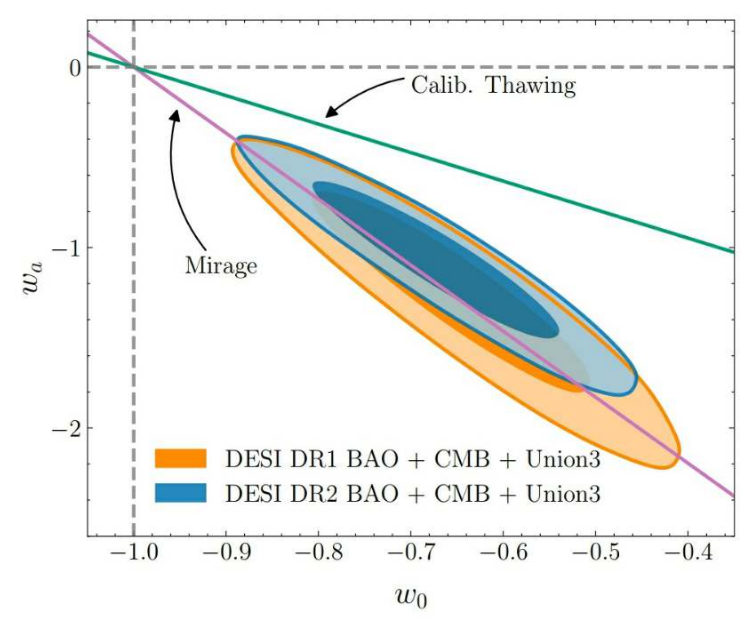

FIG. 1. Constraints on the parameters $w_0 w_a$ from DESI BAO DR2, CMB, and Union3 are illustrated in blue, while the corresponding combination with DESI BAO DR1 is shown in orange. The green line indicates the degeneracy direction associated with calibrated thawing (see Section VI A), while the purple line denotes the "mirage" direction (discussed in Section VI C) which closely follows the degeneracy direction of the $w_0 w_a$ contours.