Part A ۱۴۰۴/۱/۲۴ جبر بسم الله الرحمن الرحیم

Continuation: Dynamical aspect of expanding Universe

## Neutrino Decoupling.

To examine Neutrino decoupling, we rely on the Prior information from Particle Physics.

Also we consider that during evolution of our Univers $S$'s cts so

$$\boxed{s a^3 = cts}$$

$$\boxed{\frac{\partial}{\partial t}\left[ s a^3 \right] = -\frac{\mu}{T}\frac{d}{dt}(n a^3)}$$ Same as before ✓

with this definition $\boxed{s \equiv \frac{\rho + P - \mu n}{T}}$

Now for Relativistic Regime $T \gg m, \mu$

$$s = \frac{\rho + P - \mu n}{T} = \frac{1}{T}\sum_i \rho_i + P_i - \mu_i n_i$$

For $\mu \approx 0$

$$s \simeq \frac{1}{T}\sum \rho_i + P_i$$

$$s = \frac{2\pi^2}{45} g_* T^3$$

$$g_* = \sum_{i = \text{bosons}} g_i \left(\frac{T_i}{T}\right)^3 + 7/8 \sum_{i \text{ fermion}} g_i \left(\frac{T_i}{T}\right)^3$$

For thermal Equilibrium $T = T_i$

So

$$g_* = g_B + 7/8\, g_F$$

For Photon $n_\gamma = 2\frac{\zeta(3)}{\pi^2} T^3 \rightarrow T^3 = \frac{\pi^2 n_\gamma}{2\zeta(3)}$

$$S = \frac{2\pi^2 g_*}{45} T^3 = g_* \frac{2\pi^2}{45} \frac{\pi^2 n_\gamma}{2\zeta(3)}$$

$$= g_* \frac{\pi^4 n_\gamma}{\zeta(3)}$$

$$\underbrace{s \simeq 1.3\, g_* n_\gamma}$$

This means that number density of photons represents entropy density

☆ $$s_i = \frac{4}{3} \frac{\rho_i}{T} \begin{cases} \text{BE} & s_i = g_i \frac{2\pi^2}{45} T^3 \\ \text{FD} & s_i = \frac{7}{8} g_i \frac{2\pi^2}{45} T^3 \end{cases}$$

## 2.4.4: Neutrino decoupling.

Suppose that $s a^3 = \text{cts}$ also the main species at early Universe are

Boson : $\gamma$ — $g_B = 2$

Fermion $\quad e^- \quad e^+ \quad \nu \quad \nu^+$

$g = 2 \quad 2 \quad 3 \quad 3$ $\qquad ds \geq \frac{dQ}{T}$

Therefore $s = \frac{2\pi^2}{45}\left\{ 2 + 7/8(2+2+3+3) \right\} T^3$

$$s = \frac{43\pi^2}{90} T^3$$ entropy density of Univers.

$$s(a_1, T_1) = \frac{43\pi^2}{90} T_1^3$$ at $a_1$ all particles are thermalized with same $T_1$

after $\nu$-decoupling. $g = \begin{cases} g_B = 2 \leftarrow \gamma \\ g_F = \underbrace{3+3} \end{cases}$

At this era

$e^+e^-$ - annihilation $\rightarrow e^+ + e^- \rightarrow \gamma$

Therefore

$$S(a_2) = \frac{2\pi^2}{45}\left(2T_\gamma^3 + 7/8(6)T_\nu^3\right)$$

But we know that

$$S(a_1)\, a_1^3 = S(a_2)\, a_2^3$$

$$a_1^3 \frac{43}{\cancel{90}_{2}} \cancel{\pi^2} T_1^3 = \frac{2\cancel{\pi^2}}{\cancel{45}}\left(2T_\gamma^3 + \tfrac{7}{8}(6)\, T_\gamma^3\right) a_2^3$$

$$(a_1 T_1)^3 \frac{43}{2} = 4\left[\left(\frac{T_\gamma}{T_\nu}\right)^3 + \tfrac{7}{8}(6)\right](T_\nu a_2)^3$$

For Neutrino $T_\nu a = cts \Rightarrow \boxed{T_\nu a_2 = T_1 a_1 / a_2}$

$$(a_1 T_1)^3 \frac{43}{2} = 4\left[\left(\frac{T_\gamma}{T_\nu}\right)^3 + \tfrac{7}{8}(6)\right]\left(T_1 \frac{a_1 \cancel{a_2}}{\cancel{a_2}}\right)^3$$

$$\frac{T_\nu}{T_\gamma} = \left(\frac{4}{11}\right)^{1/3}$$

(A) This means that the ratio is independent from (a)

For Present $T_\nu^\circ = \left(\frac{4}{11}\right)^{1/3} T_\gamma^\circ \sim 1.95\ K$

(B) For Relativistic Regime

Three generation.

$$\rho_\nu = \frac{7}{8} g_\nu \frac{\pi^2}{30} T_\nu^4$$

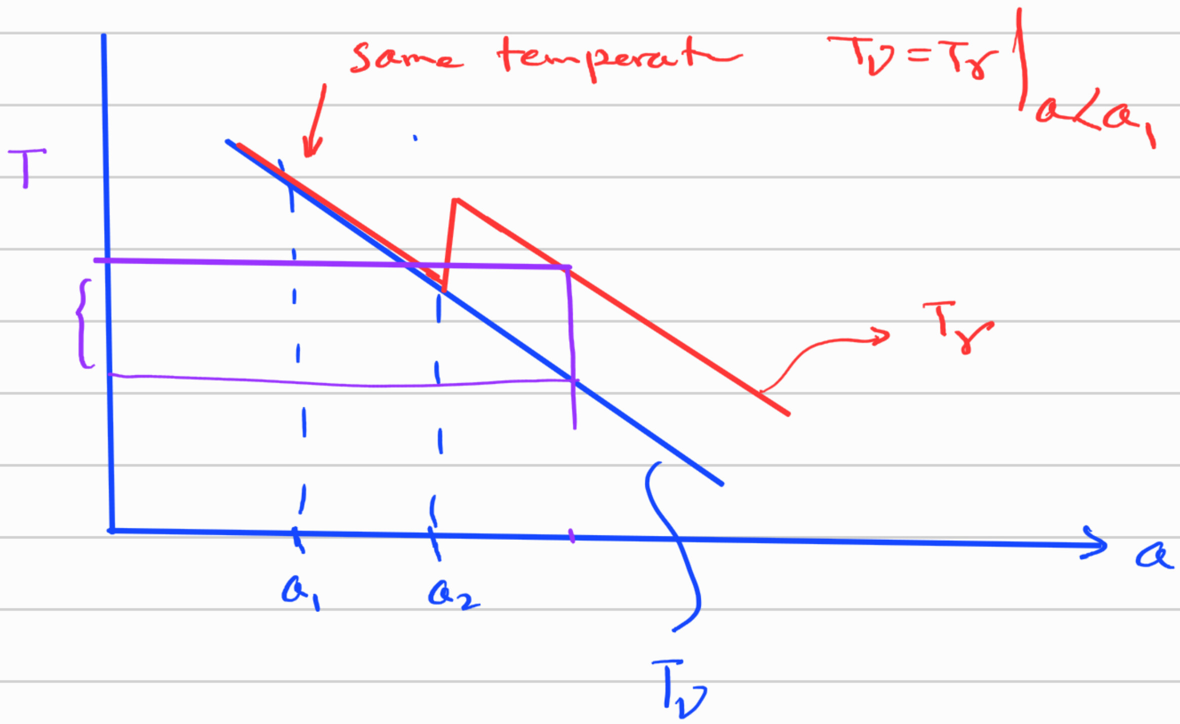
$\nu, \bar{\nu}$

$$= \frac{7}{8}(2\times3)\frac{\pi^2}{30} T_\gamma^4 \left(\frac{4}{11}\right)^{4/3}$$

$$\rho_\gamma = \frac{\pi^2}{30} 2 T_\gamma^4$$

$$\boxed{\rho_\nu = \frac{7}{8}\, 3 \left(\frac{4}{11}\right)^{4/3} \rho_\gamma} \simeq 0.7 \rho_\gamma$$

for $m_\nu \neq 0$ $\qquad \Omega_\nu^0 = \frac{\rho_\nu^0}{\rho_c^0} \sim \frac{1.68\times10^{-5}}{h^2}$



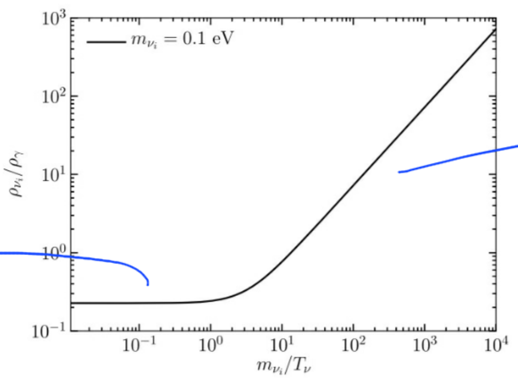
## What about massive Neutrino.

$$\rho_{\nu_i} = 2\int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{p/T}+1} \sqrt{p^2+m_\nu^2}$$

at High temper

$\rho_\nu \sim T^4 \sim a^{-4}$

$\rho_\gamma \sim T^4 \sim a^{-4}$




For low temper

$\rho_\nu \sim a^{-3}$

$\rho_\gamma \sim a^{-4}$

**FIGURE 2.5** Energy density of one generation of massive neutrinos relative to the energy density of the photons. At high temperatures, the ratio is a fixed constant; at low temperatures, the neutrino behaves like nonrelativistic matter (scaling as $a^{-3}$) and so begins to dominate over the photon density (which scales as $a^{-4}$).

## Matter- Radiation Equality

① $\rho_\gamma = 2\frac{\pi^2}{30} T_\gamma^4 \rightarrow \Omega_\gamma^0 h^2 = 2.47\times10^{-5}$

$$\Omega_\gamma(a) = \Omega_\gamma^0 a^{-4}$$

$$\Omega_m(a) = \Omega_m^0 a^{-3}$$

$$\Omega_\gamma(a=a_{eq}) = \Omega_m(a=a_{eq})$$

$$\frac{2.47\times10^{-5}}{h^2} a_{eq}^{-4} = \Omega_m^0 a_{eq}^{-3}$$

$$\boxed{a_{eq} = \frac{2.47 \times 10^{-5}}{\Omega_m^0 h^2}}$$

$$1 + z_{eq} = 2.38 \times 10^{4}\, \Omega_m^0 h^2$$

| Parameter | Symbol | Best-Fit Value (±1σ) | Data Source(s) |
|---|---|---|---|
| **Hubble constant** | $H_0$ | $67.4 \pm 0.5$ km/s/Mpc | Planck (2024) + DESI (2024) |
| **Matter density** | $\Omega_m$ | $0.315 \pm 0.007$ | Planck + DESI + Pantheon+ |
| **Dark energy density** | $\Omega_\Lambda$ | $0.685 \pm 0.007$ | Planck + DESI |
| **Baryon density** | $\Omega_b$ | $0.0493 \pm 0.0006$ | Planck (TT,TE,EE+lowE) |
| **Dark matter density** | $\Omega_c$ | $0.265 \pm 0.006$ | Planck + LSS |
| **Spatial curvature** | $\Omega_k$ | $-0.001 \pm 0.002$ (flatness consistent) | Planck + BAO |

$\Omega_r$ $\quad 9.2 \times 10^{-5}$

| Parameter | Symbol | Best-Fit Value (±1σ) |
|---|---|---|
| **Age of the universe** | $t_0$ | $13.797 \pm 0.023$ billion years |
| **Sound horizon at drag epoch** | $r_d$ | $147.18 \pm 0.29$ Mpc |
| **CMB temperature** | $T_0$ | $2.7255 \pm 0.0006$ K |

$$1 + z_{eq} = \frac{0.315}{9.2 \times 10^{-5}} \longrightarrow \boxed{z_{eq} \simeq 3400 \pm 50}$$

For ΛCDM

# Precision Detection of the Cosmic Neutrino Background

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## ABSTRACT

In the standard Big Bang cosmology the canonical value for the ratio of relic neutrinos to CMB photons is 9/11. Within the framework of the Standard Model of particle physics there are small corrections, in sum about 1%, due to slight heating of neutrinos by electron/positron annihilations and finite-temperature QED effects. We show that this leads to changes in the predicted cosmic microwave background (CMB) anisotropies that might be detected by future satellite experiments. NASA's MAP and ESA's PLANCK should be able to test the canonical prediction to a precision of 1% or better and could confirm these corrections.



# Cosmic Neutrino Last Scattering Surface

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Neutrinos decoupled from the rest of the cosmic plasma when the Universe was less than one second old, far earlier than the photons, which decoupled at $t = 380,000$ years. Surprisingly, though, the last scattering surface of massive neutrinos is much closer to us than that of the photons. Here we calculate the properties of the last scattering surfaces of the three species of neutrinos.

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## I. INTRODUCTION

The standard cosmological model predicts that neutrinos were produced in the early universe and are present today with an abundance[1] of 112 $\mathrm{cm}^{-3}$ per species [3, 4]. Detecting this background remains a tantalizing experimental dream [5, 6, 7, 8, 9, 10, 11], with recent developments encouraging the optimists [12, 13, 14, 15, 16, 17, 18].

For the time being, the question of where the cosmic neutrinos come from remains an academic question. Yet it is of sufficient interest that, even if there were no chance of detection, the origin of the cosmic neutrino background (CNB) seems worthy of theoretical study.

The neutrino *Last Scattering Surface* (LSS) is typically thought of as being located a given distance from us with a small but finite width, similar to the last scattering surface of the cosmic microwave background (CMB). Neutrinos last scatter when the temperature of the universe was a few MeV and the universe was less than a second old, while the photons in the CMB last scattered much later when the temperature was 1/3 eV at $t = 380,000$ years, so it is natural to assume that the neutrino LSS is further away than that of the CMB. Calculating how much further away leads to a little surprise. For, even massless particles can travel only a very small comoving distance in the very early universe, so the CMB comes from a comoving distance about 9540 $h^{-1}$ Mpc away from us (where the present expansion rate is $H_0 = 100\,h$ km $\mathrm{s}^{-1}$ $\mathrm{Mpc}^{-1}$), while *massless* neutrinos arrive from a comoving distance 9735 $h^{-1}$ Mpc away. That is, neutrinos travel only about 200 Mpc (comoving) in the first 380,000 years.

sive neutrinos [19, 20]: the CNB actually reaches us from *closer* than the CMB. Even for neutrino masses as small as 0.05 eV (and one of the neutrinos must be at least this massive) the effect is dramatic. For neutrinos with mass of 1 eV, the effect is truly striking with most of these neutrinos arriving from only several *hundred* Mpc away!

## II. LAST SCATTERING SURFACE OF MASSIVE NEUTRINOS

Neutrinos stopped scattering when temperatures were of order a few MeV and the universe was less than a second old. At that time each neutrino species had a Fermi-Dirac distribution for a massless particle (assuming – as we will throughout – zero chemical potential and no heating from electron-positron annihilation).

Consider first the calculation of the last scattering surface of a massless neutrino. The comoving distance travelled by a massless particle starting from $t_i = 1$ sec until today is

$$\chi = \int_{t_i}^{t_0} \frac{dt}{a(t)} = \int_{a_i}^{1} \frac{da}{a^2 H(a)}. \qquad (1)$$

Note that early on $H(a) = H_0 \Omega_r^{1/2} a^{-2}$ where $\Omega_r = 8.3 \times 10^{-5}$ is the radiation density *today* in units of the critical density. The integrand peaks at late times, so the contribution to the comoving distance from early times ($t_i \sim 1$ sec) is negligible; this explains our nonchalance in defining the initial time. It also explains why all three species of neutrinos share the same last scattering surface even though electron neutrinos decouple slightly later than do the other two species. The comoving distance