بسم الله الرحمن الرحیم

جلسه ۱۹ / ۱ / ۱۴۰۴

3 Continuation: Statistical Mechanics of Ideal Gas in expanding Universe

In thermodynamical Equilibrium

$$\begin{cases} \langle n_k \rangle^{MB} = e^{-\beta(\epsilon_k - \mu)} \equiv f \\ \langle n_k \rangle^{BE,FD} = \dfrac{1}{e^{\beta(\epsilon_k - \mu)} \mp 1} \equiv f \end{cases}$$

$$\left\{ \hbar = 1,\ K_B = 1,\ C = 1 \right\}$$

Collisional terms

$$\frac{df}{dt} = C[f] = 0$$

$$\begin{cases} U = \langle H \rangle = \sum_{k=1}^{\infty} \langle n_k \rangle \epsilon_k \\ N = \sum_{k=1}^{\infty} \langle n_k \rangle \end{cases}$$

In Continuum Regime: $\sum_k \longrightarrow \int \frac{d^3k}{\left(\frac{2\pi}{L}\right)^3} = \int d\epsilon\, g(\epsilon)$ $\boxed{N, V \to \infty}$

$g(\epsilon)$: DOS

$$g(\epsilon)\, d\epsilon = g(k)\, dk = g_s \frac{4\pi k^2}{\left(\frac{2\pi}{L}\right)^3} dk$$

g_s: Degree of freedom for Spin / Polarization

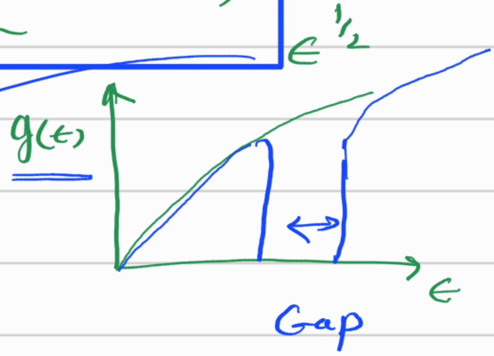
$$g(k) = \frac{4\pi k^2}{\left(\frac{2\pi}{L}\right)^3} = \frac{d^2k}{\left(\frac{2\pi}{L}\right)^3}$$

Number Density of Particles

$$\star \quad n \equiv \frac{N}{V} = \frac{1}{V} \sum_k \langle n_k \rangle = \frac{1}{\cancel{V}} \int \frac{d^3k}{\left(\frac{2\pi}{\cancel{L}}\right)^3} \langle n_k \rangle$$

☆ $$\boxed{n = \int \frac{d^3k}{(2\pi)^3} \langle n_k \rangle = \int d\epsilon \, g(\epsilon) \, \langle n(\epsilon) \rangle}$$

$$\boxed{\mathcal{H} = p^2/2m}$$



$$\langle \mathcal{H} \rangle = \sum_k \langle n_k \rangle \epsilon_k$$

$$\rho = \frac{\langle \mathcal{H} \rangle}{V} = \frac{1}{V} \sum_k \langle n_k \rangle \epsilon_k \longrightarrow \text{Continuum Regime}$$

ρ: Energy Density

$\downarrow \begin{Bmatrix} N \to \infty \\ V \to \infty \end{Bmatrix}$

☆ $$\boxed{\rho = \int \frac{d^3k}{(2\pi)^3} \langle n_k \rangle \epsilon_k = \int d\epsilon \, g(\epsilon) \, \langle n(\epsilon) \rangle \, \epsilon}$$

☆ What about Pressure $P = ?$

$$\frac{PV}{k_B T} = \frac{1}{a} \sum_{k=1}^{\infty} \ln(1 + az\,e^{-\beta\epsilon_k})$$

$$\boxed{p = \hbar k}$$

$$\frac{dp}{\hbar} = dk \to 1$$

$$\lim_{N,V\to\infty} \frac{PV}{k_B T} = \frac{V}{a} \int \frac{d^3k}{(2\pi)^3} \ln(1 + az\,e^{-\beta\epsilon_k})$$

$$\boxed{\frac{PV}{k_B T} = \frac{V}{a} \int d^3p \, \ln(1 + az\,e^{-\beta\epsilon_k})}$$

Integration by Part

$$\boxed{P = \frac{1}{3}\frac{N}{V}\left\langle p\frac{d\epsilon}{dp}\right\rangle} = \underbrace{\frac{1}{3}n\left\langle p\frac{d\epsilon}{dp}\right\rangle}_{\text{General Case}}$$

To Know more See below from Chapter 6.4 Pathria

$$P = \frac{kT}{a}\int_0^\infty \ln\left[1 + aze^{-\beta\varepsilon(p)}\right]\frac{4\pi p^2 dp}{h^3}$$

$$= \frac{4\pi kT}{ah^3}\left[\frac{p^3}{3}\ln\left[1 + aze^{-\beta\varepsilon(p)}\right]\Big|_0^\infty + \int_0^\infty \frac{p^3}{3}\frac{aze^{-\beta\varepsilon(p)}}{1 + aze^{-\beta\varepsilon(p)}}\beta\frac{d\varepsilon}{dp}dp\right].$$

$$P = \frac{4\pi}{3h^3}\int_0^\infty \frac{1}{z^{-1}e^{\beta\varepsilon(p)} + a}\left(p\frac{d\varepsilon}{dp}\right)p^2 dp.$$

$$N = \int \langle n_p\rangle \frac{Vd^3p}{h^3} = \frac{4\pi V}{h^3}\int_0^\infty \frac{1}{z^{-1}e^{\beta\varepsilon(p)} + a}p^2 dp.$$

$$P = \frac{1}{3}\frac{N}{V}\left\langle p\frac{d\varepsilon}{dp}\right\rangle = \frac{1}{3}n\langle pu\rangle,$$

For photons $\quad \epsilon = pc \longrightarrow p\frac{d\epsilon}{dp} = \epsilon$

$$P = \frac{1}{3}n\left\langle p\frac{d\epsilon}{dp}\right\rangle = \frac{1}{3}\widetilde{n\epsilon} = \frac{1}{3}\rho$$

For Classical Particles $\quad \epsilon = \frac{p^2}{2m} \longrightarrow p\frac{d\epsilon}{dp} = 2\epsilon$

$$P = \frac{1}{3} n \langle p \frac{d\epsilon}{dp} \rangle = \frac{1}{3} n \, 2\epsilon = \frac{2}{3} n \epsilon = \frac{2}{3} \rho$$

For Dark Matter $\longrightarrow$ Cold Dark Matter

$$\langle v^2 \rangle \ll c^2$$

$$P = \underbrace{n}_{\sim} K_B T = \frac{\rho}{\mu c^2} \underbrace{K_B T}_{\downarrow}$$

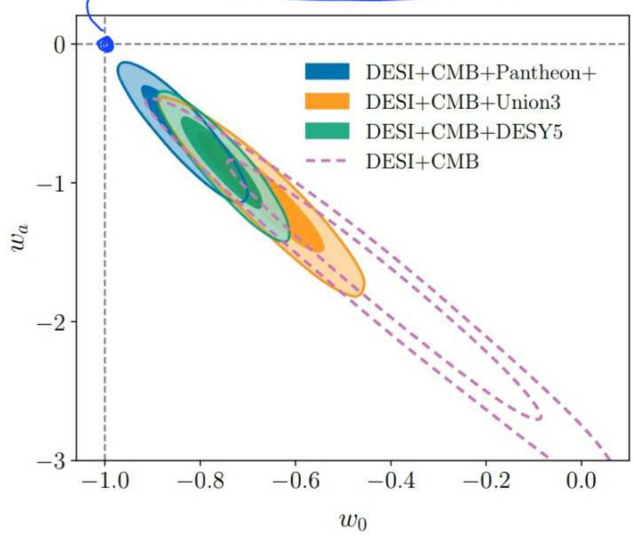
$$\simeq \frac{\rho}{\mu c^2} \langle v^2 \rangle \mu$$

$$P \sim \frac{\rho \langle v^2 \rangle}{c^2} \sim 0$$

$$P \sim 0 \longrightarrow \boxed{w = 0}$$

For <u>Dark Energy</u> $\quad P_\Lambda = -\frac{\Lambda}{8\pi G} = -\rho_\Lambda$

Cosmological



$$W_\Lambda = -1$$

FIG. 11. Results for the posterior distributions of w_0 and w_a, from fits of the $w_0 w_a$CDM model to DESI in combination with CMB and three SNe datasets as labelled. We also show the contour for DESI combined with CMB alone. The contours enclose 68% and 95% of the posterior probability. The gray dashed lines indicate $w_0 = -1$ and $w_a = 0$; the ΛCDM limit ($w_0 = -1$, $w_a = 0$) lies at their intersection. The significance of rejection of ΛCDM is 2.8σ, 3.8σ and 4.2σ for combinations with the Pantheon+, Union3 and DESY5 SNe samples, respectively, and 3.1σ for DESI+CMB without any SNe.

$$W = w_0 + w_a (1 - a)$$

$$W_\Lambda = w_0 = -1 \quad , \quad \boxed{\begin{matrix} w_a = 0 \\ w_0 = -1 \end{matrix}}$$

For more Information about this tension check:

DESI DR2 Results II: Measurements of Baryon Acoustic Oscillations and Cosmological Constraints

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Extended Dark Energy analysis using DESI DR2 BAO measurements

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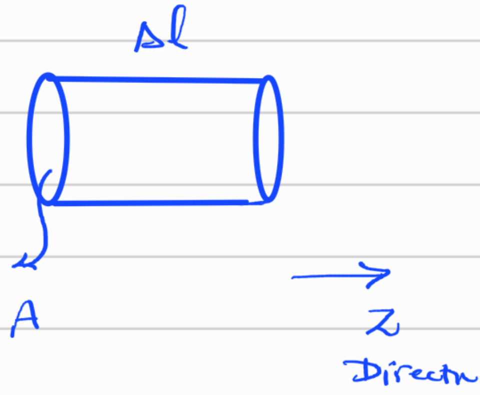
To compute Pressure, we can also use another approach such that:

$$dP = \frac{n A \Delta l \, \frac{\Delta p_z}{\Delta t}}{A}$$



For elastic collision $\Delta p_z = 2p_z$

$$dP = n \frac{\Delta l}{\Delta t} 2p_z = n\, 2\underbrace{p_z}_{mv_z} v_z$$

$$P = \int dP = 2nm \int d\bar{v}\, v_z^2 \underbrace{P(v)}_{\text{Velocity Distribution}} \theta(v_z)$$

$$P = nm\langle v_z^2 \rangle = \frac{1}{3} nm \langle v^2 \rangle = \frac{1}{3} \frac{\rho}{c^2} \langle v^2 \rangle$$

$$P = \frac{1}{3} n \left\langle p \frac{d\epsilon}{dp} \right\rangle$$

④ Thermodynamics of expanding Universe

$$f^{BE,FD}(\vec{p}) = \frac{1}{(2\pi)^3}\frac{1}{e^{\beta(\epsilon(p)-\mu)} \mp 1}$$

$$\epsilon(p) = \sqrt{p^2 + m_0^2}$$

$$f^{MB}(\vec{p}) = \frac{1}{(2\pi)^3} e^{-\beta(\epsilon(p)-\mu)}$$

Number density of species ith

$$n_i = g_i \int d^3p \, f(p,T)$$

$$\boxed{n_i = g_i \, 4\pi \int d\epsilon \sqrt{\epsilon^2 - m_0^2} \, f(\epsilon,T)}$$

Energy Density

$$\rho_i = g_i \int d^3p \, f(p,T) \, \epsilon(p) = g_i \int \frac{d^3k}{(2\pi)^3} \langle n(k) \rangle \epsilon(k)$$

$$\boxed{\rho_i = g_i 4\pi \int d\epsilon \, \epsilon \sqrt{\epsilon^2 - m_0^2} \, f(\epsilon,T) \, \epsilon}$$

Pressure

$$P_i = \frac{1}{3} n_i \left\langle p \frac{\partial \epsilon_i}{\partial p} \right\rangle = \frac{g_i \cdot 4\pi}{3} \int p^2 dp \, f(p,T) \, p \frac{\partial \epsilon_i}{\partial p}$$

$$= \frac{4\pi g_i}{3} \int d\epsilon \, \epsilon \sqrt{\epsilon^2 - m_\circ^2} \, f(\epsilon,T) \sqrt{\epsilon^2 - m_\circ^2} \, \frac{\sqrt{\epsilon^2 - m_\circ^2}}{\epsilon}$$

$$\boxed{P_i = \frac{4\pi g_i}{3} \int d\epsilon \, (\epsilon^2 - m_\circ^2)^{3/2} f(\epsilon,T)}$$

☐ At Relativistic Regime $\quad T \gg m_i, \; \mu \to 0$

For FD

$$n_i = \frac{3}{4} g_i \frac{\zeta(3)}{\pi^2} T^3$$

$$\rho_i = \frac{7}{8} g_i \frac{\pi^2}{30} T^4$$

$$P_i = \frac{1}{3} \rho_i \quad , \quad \boxed{\frac{\rho_i}{n_i} = 3.15\, T}$$

For BE

$$n_i = g_i \frac{\zeta(3)}{\pi^2} T^3$$

$$\rho_i = g_i \frac{\pi^2}{30} T^4$$

$$P_i = \frac{1}{3} \rho_i \qquad \boxed{\frac{\rho_i}{n_i} = 2.7\, T}$$

For γ $\quad \rho_\gamma = 2 \frac{\pi^2}{30} T_\gamma^4 \qquad \Omega_\gamma^\circ = \frac{\rho_\gamma^\circ}{\rho_c^\circ} = \frac{2.47 \times 10^{-5}}{h}$

$3H_\circ^2/8\pi G \leftarrow \rho_c^\circ$

$$\boxed{H_0 = 100h}$$

□ Entropy

First approach : $\left\{ dE = T dS - P dV + \mu dN \right.$

$$\left\{ \begin{array}{l} S = s a^3 \\ V \sim a^3 \\ E = \rho a^3 \end{array} \right.$$

$$\boxed{s = \frac{\rho + P - \mu n}{T}}$$

$$T d(s a^3) = d(\rho a^3) + P da^3 - \mu d(n a^3)$$

From continuity Eq.

$$\frac{d\rho}{dt} + \frac{3\dot{a}}{a}(\rho + P) = 0 \implies \boxed{d(\rho a^3) = -P da^3}$$

$$T d(s a^3) = -\cancel{P da^3} + \cancel{P da^3} - \mu d(n a^3)$$

$$\boxed{d(s a^3) = -\frac{\mu}{T} d(n a^3)}$$

For $\frac{\mu}{T} \to 0 \longrightarrow s a^3 = cts$

For $n a^3 = cts \longrightarrow s a^3 = cts$

Second approach:

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0 \implies a^{-3}\frac{\partial}{\partial t}\left[(\rho + P)a^3\right] - \frac{\partial P}{\partial t} = 0$$

$$a^{-3}\frac{\partial}{\partial}\left((\rho + P)a^3\right) - \frac{\partial P}{\partial T}\frac{\partial T}{\partial t} = 0$$

$$P = \frac{g_i}{6\pi^2}\int d\epsilon \frac{(\epsilon^2 - m_0^2)^{3/2}}{e^{(\epsilon - \mu_i)\beta} \mp 1}$$

$$\frac{dP}{dT} = \frac{\rho + P}{T} - nT\frac{d}{dT}\left(\mu/T\right)$$

$$a^{-3}\frac{\partial}{\partial t}\left[(\rho + P)a^3\right] - \left(\frac{\rho + P}{T} + nT\frac{d}{dT}\left(\mu/T\right)\right)\frac{\partial T}{\partial t} = 0$$

$$a^{-3}T\frac{\partial}{\partial t}\left[\frac{(\rho + P)a^3}{T} - \frac{\mu n a^3}{T}\right] = -\mu a^{-3}\frac{d}{dt}(na^3)$$

$$a^{-3}T\frac{\partial}{\partial t}\left[\left(\frac{\rho + P - \mu n}{T}\right)a^3\right] = -\mu a^{-3}\frac{d}{dt}(na^3)$$

$$\frac{d}{dt}(sa^3) = -\frac{\mu}{T}\frac{d}{dt}(na^3)$$

For Relativistic Regime

$$s = \frac{2\pi^2}{45}g_* T^3$$

$$g_* = \sum_{i's\ boson} g_i\left(\frac{T_i}{T}\right)^3 + 7/8\sum_{i's\ Fermion} g_i\left(\frac{T_i}{T}\right)^3$$

For thermal Equilibrium $\boxed{T_i = T}$

$$g_* = g_B + 7/8\, g_F \qquad g_B \equiv \sum_i g_i^B, \quad g_F \equiv \sum_i g_i^F$$

For photon $n_\gamma = \frac{2\zeta(3)}{\pi^2} T_\gamma^3 \Rightarrow \boxed{T_\gamma^3 = \frac{n_\gamma \pi^2}{2\zeta(3)}}$

$$s = \frac{2\pi^2 g_*}{45} T^3 = g_* \frac{2\pi^2 \, \pi^2 n_\gamma}{45 \; 2\zeta(3)}$$

$$\boxed{s = \frac{g_* \pi^4 n_\gamma}{45\zeta(3)} \simeq 1.3\, g_* n_\gamma}$$

$$s_i = \frac{4}{3}\frac{\rho_i}{T} \begin{cases} \text{FD} & s_i = \frac{7}{8} g_i \frac{2\pi^2}{45} T^3 \\ \text{BE} & s_i = g_i \frac{2\pi^2}{45} T^3 \end{cases}$$

2.4.4 Neutrino decoupling $\longrightarrow \boxed{\frac{T_\nu}{T_\gamma} = \left(\frac{4}{11}\right)^{1/3}}$? ?

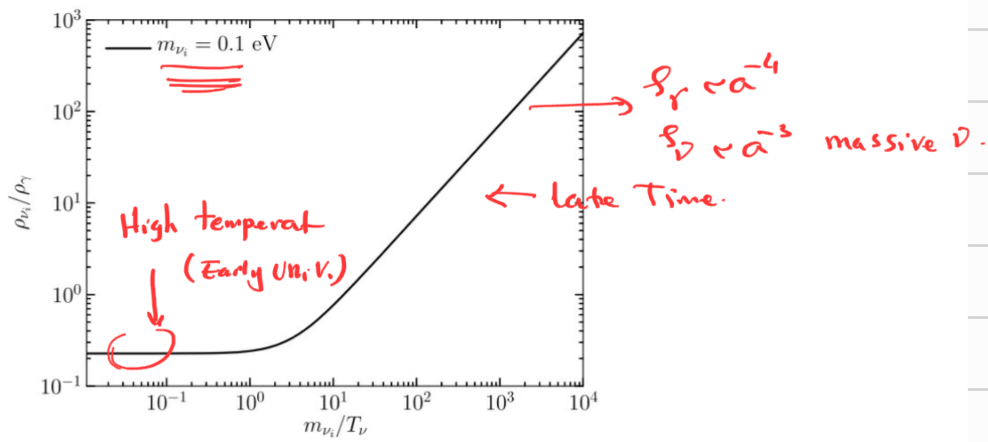


FIGURE 2.5 Energy density of one generation of massive neutrinos relative to the energy density of the photons. At high temperatures, the ratio is a fixed constant; at low temperatures, the neutrino behaves like nonrelativistic matter (scaling as a^{-3}) and so begins to dominate over the photon density (which scales as a^{-4}).