بسم الله الرحمن الرحیم

جلسه ۱۴۰۳/۱۲/۲۶

\# Background of our Cosmos → Geometrical Criteria
→ Background Evolution

- ①,② Physical length
- Comoving length
- ③ Angular diameter distance
- ④ Luminosity distance
- Comoving volume element
- Cosmic age

$$R(t) = a(t)\chi$$

$$ds^2 = c^2 dt^2 - a(t)^2\left[d\chi^2 + S(\chi)\, d\Omega^2\right]$$

Null Geodesic →
$$\begin{cases} ds^2 = 0 \\ d\Omega^2 = 0 \end{cases} \to d\chi = \frac{cdt}{a(t)} \to \chi = c\int_t^{t_0}\frac{dt'}{a(t')} = c\int_0^z \frac{dz}{H(z)}$$

$$\begin{cases} ds^2 = 0 \\ d\chi^2 = 0 \end{cases} \to a(t)S(\chi)\theta = \Delta l \to d_A \equiv a(t)S(\chi)$$

$$d_l \equiv (1+z)S(\chi) = d_A(1+z)^2$$

$$f \equiv \frac{dV}{d\Omega dz} = \frac{S(\chi)^2}{H(z)} \to \frac{dN}{dz} = n(z)\frac{dV}{dz} = 4\pi n(z)\frac{S(\chi)^2}{H(z)}$$

$$\Delta N_{L>L_{\min}} = \int_{L_{\min}}^{\infty}\frac{dN}{dzdl}dl\Delta z$$

$$t_0 = \int_0^{t_0} dt = \int_0^{\infty}\frac{dz}{(1+z)H(z)}$$

مهمترین کمیتهای رصدی

گروه اول: این گروه شامل کمیت هایی است که تحول زمینه کیهان را مشخص می کنند

$$\Omega_m, \Omega_\nu, \Omega_b, \Omega_K, \Omega_\lambda, w, t_0, H_0, q_0, T_{CMB}$$

گروه دوم: مشخص کننده انحراف از همگنی و همسانگردی است

$$\sigma_8, A_s, A_t, n_s, n_t, dn/d\ln k$$

$$\Omega_m = \Omega_{DM} + \underbrace{\Omega_b + \Omega_\nu}$$

$$H(z) = H_0^2\left[\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda(1+z)^{3(1+w)}\right]$$

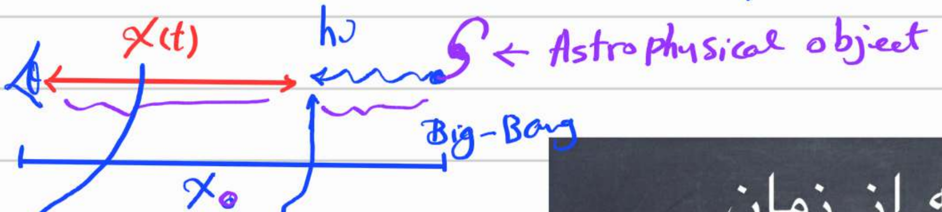
$$\chi = c\int_0^z \frac{dz'}{H(z)} \qquad \frac{\dot a}{a} = H_{DM}^2 = H_0\,\Omega_m(1+z)^3 \rightarrow a_{DM} \sim t^{2/3}$$

Background Quantities (Continuation)

1. Physical and Comoving Distance

2. Photon arrival path

اگر در انفجار بزرگ همه چیز در کنار هم بوده است پس چرا فوتونی که در افق ذره در انفجار بزرگ قرار گرفته به اندازه سن عالم وقت نیاز دارد که به ما برسد؟



$\boxed{\Omega_r = 0 \ ,\ \Omega_\Lambda = 0}$

مسیر فوتونی که از زمان
انفجار اولیه تا زمان حال طی

سوال از این قرار است که اگر در زمان انفجار اولیه هر چیزی دقیقا در کنار هم باشند پس چرا تقریبا به اندازه سن عالم طول می کشد تا این فوتونهای اولیه به ما برسند؟

$$\chi(t) = A \equiv c\int_0^t \frac{dt'}{a(t')} = \chi_0 - \chi(t) \qquad a(t) = \left(\frac{t}{t_0}\right)^{2/3}$$

فرض کنیم دوران غالب بودن ماده است

$$A = +3ct_0\left(\frac{t}{t_0}\right)^{1/3} \qquad \chi(t) = \chi_0 - 3ct_0\left(\frac{t}{t_0}\right)^{1/3}$$

$$\chi_0 = c\int_0^{t_0}\frac{dt'}{a(t')} = 3ct_0$$

$$R(t) = a(t)\chi(t) = 3ct_0\left[\left(\frac{t}{t_0}\right)^{2/3} - \left(\frac{t}{t_0}\right)\right]$$

Matter Dominated era

$$☆\ \chi(t) = \chi_0 - \int_0^t \frac{c\,dt'}{a(t')}\ ☆$$

$$\chi_0 = \int_0^{t_0}\frac{c\,dt'}{a(t')} = c\int_0 dt'\left(\frac{t'}{t_0}\right)^{-2/3} \qquad \sim \left(\frac{t}{t_0}\right)^{2/3}$$

$$\boxed{\chi_0 = 3ct_0}$$

and

$$\int_0^t c\,dt'\left(\frac{t'}{t_0}\right)^{-2/3} = 3ct_0\left(\frac{t}{t_0}\right)^{1/3}$$

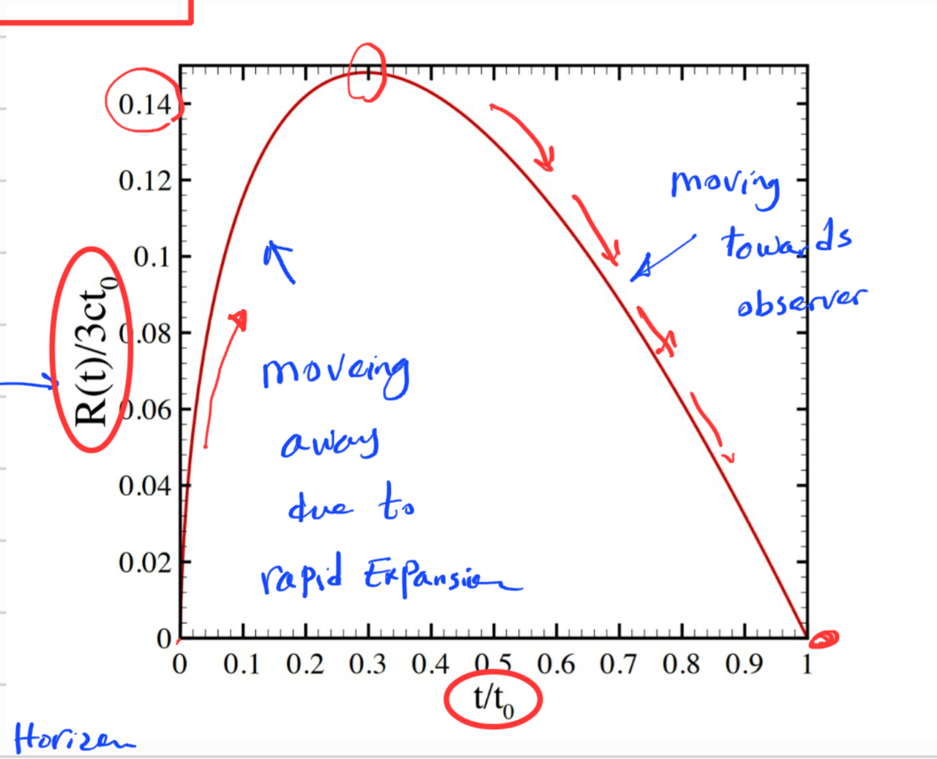
So

$$\chi(t) = 3ct_0 - 3ct_0\left(\frac{t}{t_0}\right)^{1/3} = \chi_0 - c\int_0^t \frac{dt'}{a(t')}$$

$\longrightarrow$ therefore physical lenght is

$$R(t) = a(t)\,\chi(t) = \left(\frac{t}{t_0}\right)^{2/3}\left[3ct_0 - 3ct_0\left(\frac{t}{t_0}\right)^{1/3}\right]$$

$$\boxed{R(t) = 3ct_0\left[\left(\frac{t}{t_0}\right)^{2/3} - \left(\frac{t}{t_0}\right)\right]}$$

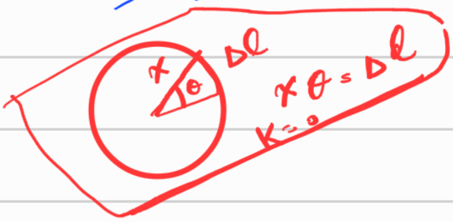


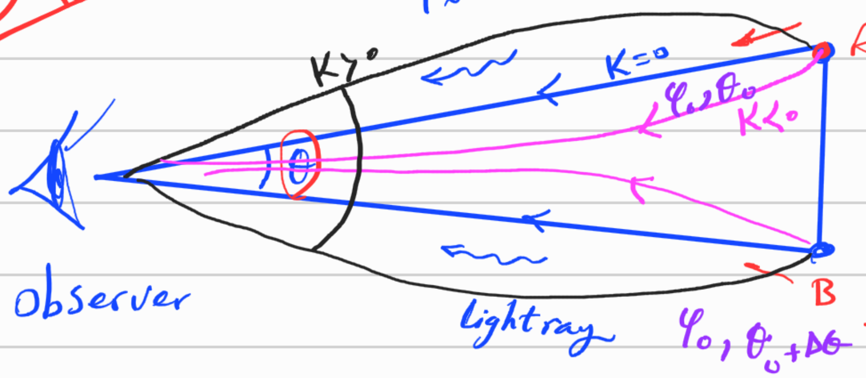
physical Path passed by photon from a source located at Horizon

③ Angular Diameter Distance.

داستان چیست؟ : در واقع به دنبال پاسخ به این سوال هستیم که یک جسم کوچک توسط ناظر چگونه دیده می‌گردد.

که با استفاده از آن به سایر موارد نظیر درخشندگی .. برسیم





$$\Delta l_s = a(t_s)\,\Delta X_s$$

↑ Physical length

$$\rightarrow ds^2 = c^2 dt^2 - a^2(t)\left[d\chi^2 + S(\chi)^2 d\Omega^2\right]$$

Null-Geodesic $ds=0$ and $d\chi = 0$, $d\varphi = 0$

Two rays traveling from A and B

$$\theta\, S(\chi) = \Delta X_s$$ براساس شکل خواهیم داشت :

طول فیزیکی عمود بر راستای دید

$$d_A \equiv \frac{\Delta l_s}{\theta} = \frac{a(t_s)\,\Delta X_s}{\theta} = \frac{a(t_s)\,\cancel{\theta}\, S(\chi)}{\cancel{\theta}}$$

$$\boxed{d_A = a_s\, S(\chi)}$$ Angular Diameter Distance

$$\boxed{H = \mathcal{H} H_0}$$

$$d_A = \frac{c}{(1+z)\, H_0 \sqrt{|k|}}\, \widetilde{\mathrm{Sinn}}\left[\sqrt{|k|}\int_0^z \frac{dz'}{\mathcal{H}(z)}\right]$$

Recall that
$$\begin{cases} S(\chi) = \dfrac{1}{\sqrt{|k|}}\,\mathrm{Sinn}\left[\sqrt{|k|}\,\chi\right] \\[2ex] \chi = \dfrac{c}{H_0}\displaystyle\int_0^z \frac{dz'}{\mathcal{H}(z')} \end{cases}$$

$$\mathrm{Sinn} \equiv \begin{cases} 1 & K=0 \\ \sin & K>0 \\ \sinh & K<0 \end{cases}$$

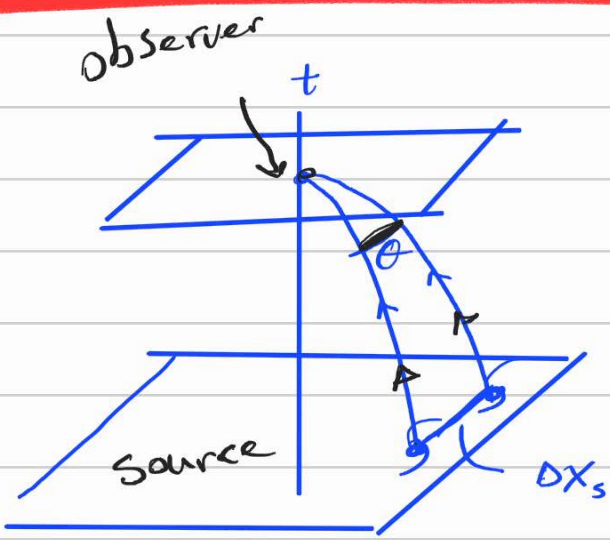
$$H^2 = H_0^2[\quad] = H_0^2\,\mathcal{H}^2$$

$$\boxed{H = H_0\,\mathcal{H}}$$

To introduce consistent quantity.

$$\boxed{S(\chi) = \frac{c}{H_0}\frac{1}{\sqrt{|k|}}\,\mathrm{Sinn}\left[\sqrt{|k|}\int_0^z \frac{dz'}{\mathcal{H}(z')}\right]}$$

Ex1:



$$d_A\,\theta = \Delta l_s = \Delta X_s\, a_s$$

$$\Delta X_s = \theta\, S(X_s)$$

$$d_A = \frac{\Delta X_s\, a_s}{\theta}$$

$$d_A = \frac{\cancel{\theta}\, S(X_s)}{(1+z_s)\,\cancel{\theta}}$$

Ex 2: Energy Flux and Luminosity Distance

یک منبع نوری در انتقال به سرخ z_s قرار دارد. ناظر شار دریافتی را به صورت زیر محاسبه می‌کند

1. $$\phi_0 = \frac{dN_0}{dt_0} = \frac{dN_0}{(1+z)\,dt_s} = \frac{dN_s}{(1+z)\,dt_s} = \frac{\phi_s}{(1+z_s)}$$

Cosmological Redshift

درخشندگی ذاتی

$$L = \text{Luminosity} = h\nu_s \phi_s$$

$$\frac{d^2E_0}{dA_0\,dt_0}$$

$$\text{Energy Flux} \equiv F_0 = \frac{h\nu_0\,\phi_0}{A_0} = \frac{h\nu_s}{(1+z_s)}\,\frac{\phi_s}{(1+z_s)}\,\frac{1}{4\pi S(\chi)^2}$$

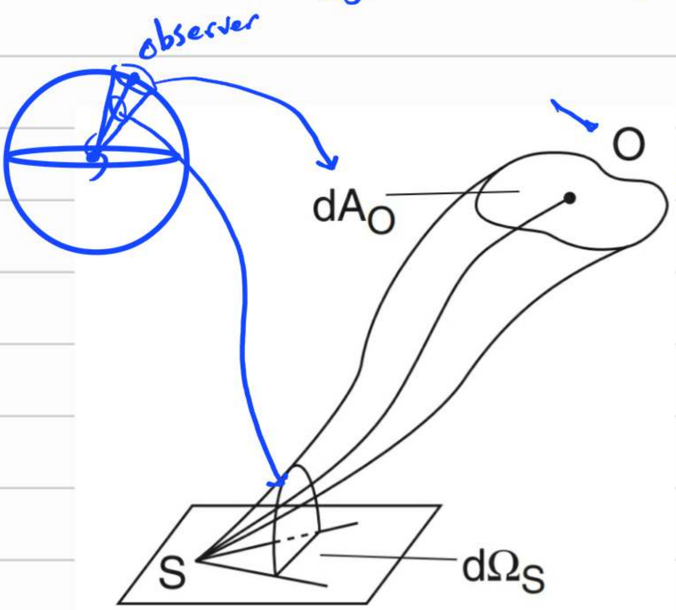


FIG. 1. A light beam emitted at the source event S ending on the observer O. At the source position, the plane normal to the source four-velocity is indicated.

$$dA_0 = d\Omega_s\, S(\chi_s)$$

Intrinsic Luminosity

$$F_0 = \frac{L}{4\pi\left[(1+z)\,S(\chi)\right]^2} = \frac{L}{4\pi d_L^2}$$

$$\boxed{d_L \equiv (1+z)\,S(\chi) = (1+z)^2 d_A}$$

$$L_B \equiv \int d\nu\, h\nu\, \phi(\nu)$$

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Fluctuations of the luminosity distance

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(Received 7 November 2005; published 27 January 2006)

We derive an expression for the luminosity distance in a perturbed Friedmann universe. We define the correlation function and the power spectrum of the luminosity distance fluctuations and express them in terms of the initial spectrum of the Bardeen potential. We present semianalytical results for the case of a pure CDM (cold dark matter) universe. We argue that the luminosity distance power spectrum represents a new observational tool which can be used to determine cosmological parameters. In addition, our results shed some light into the debate whether second order small scale fluctuations can mimic an accelerating universe.

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2. Magnitude درخشندگی

$$m = -2.5 \log_{10} F$$

$$F = \left(10^{-m}\right)^{2/5} = (100)^{-\frac{m}{5}}$$

تغییر صد برابری درخشان یعنی
Δm ۵ برابر شده است.

$$\boxed{\frac{F_2}{F_1} = 100^{\frac{m_1 - m_2}{5}}}$$

تاریخچه: قدر ستاره Vega

☆ Recall that

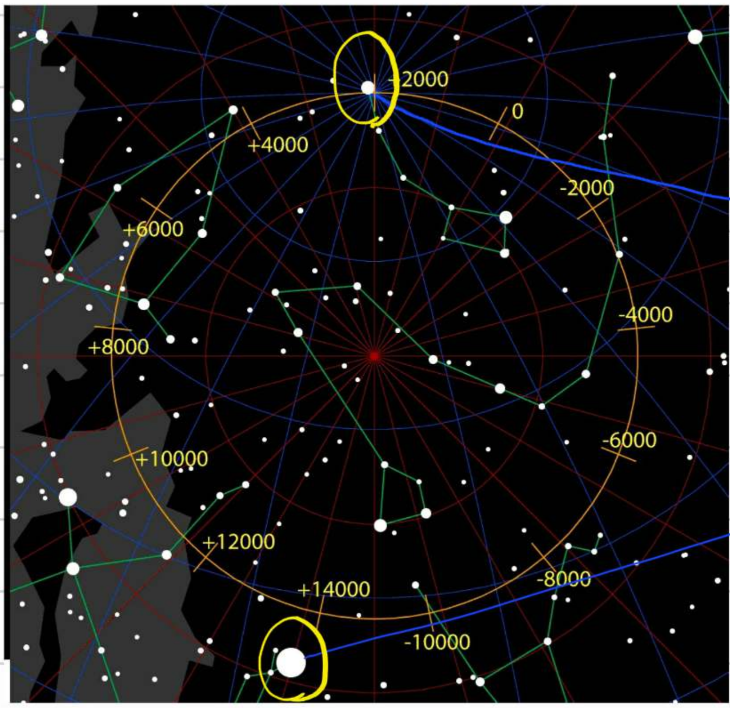
$m_{sun} = -26.8$

$m_{moon} = -12.74$

$$\frac{F_{sun}}{F_{moon}} = (100)^{\frac{-12.74+26.8}{5}}$$

$$\approx 100^{\frac{14}{5}} \sim 400.000$$

درخشان‌تر است



ستاره قطبی

Vega

$m_{Vega} = 0$

$$m = -2.5 \log F = -2.5 \log\left(\frac{L}{4\pi d_L^2}\right) = -2.5 \log L + 5 \log d_L + 2.5 \log 4\pi$$

$$m = 5 \log d_L - 2.5 \log\left(\frac{L}{4\pi}\right)$$

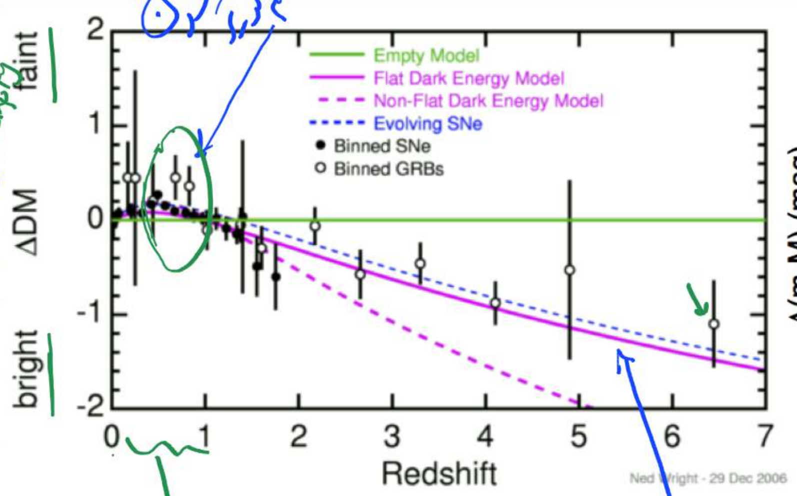
☆ براساس تعریف قدر ظاهری و قدر مطلق حسبی در فاصله 10pc یکسان

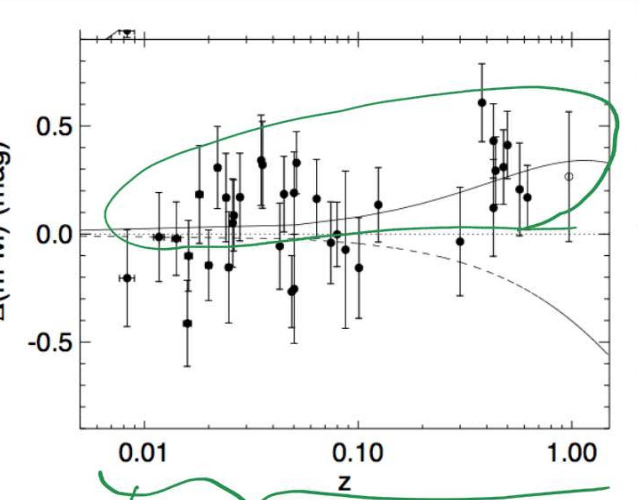
$$\mu = m - M = -2.5 \log\left(\frac{F}{F_M}\right) = -2.5 \log\left(\frac{\frac{L}{4\pi d_L^2}}{\frac{L}{4\pi (10pc)^2}}\right)$$

$$\mu = 5 \log\left(d_L(pc)\right) - 5$$

or $$\boxed{\mu = 5 \log\left(d_L(Mpc)\right) - 25}$$

$\Delta\mu = \mu_\Lambda - \mu_{empty}$





$K=0$

$$d_L = (1+z) \int_0^z \frac{dz'}{H(z')}$$

EX3: Fluctuation in z and d_L due to inhomogeniety

$$\underbrace{}_{\int_0^z \frac{dz}{H(z)}}$$

$$\tilde{d}_L(z_S, \mathbf{n}) = (1+z_S)(\eta_O - \eta_S)\Bigg\{1 - \frac{1}{(\eta_O-\eta_S)\mathcal{H}_S}\mathbf{v}_O\cdot\mathbf{n} - \left(1 - \frac{1}{(\eta_O-\eta_S)\mathcal{H}_S}\right)\mathbf{v}_S\cdot\mathbf{n}$$
$$- \left(2 - \frac{1}{(\eta_O-\eta_S)\mathcal{H}_S}\right)\Psi_S + \left(1 - \frac{1}{(\eta_O-\eta_S)\mathcal{H}_S}\right)\Psi_O$$
$$+ \frac{2}{(\eta_O-\eta_S)}\int_{\eta_S}^{\eta_O} d\eta\,\Psi + \frac{2}{(\eta_O-\eta_S)\mathcal{H}_S}\int_{\eta_S}^{\eta_O} d\eta\,\dot{\Psi} - 2\int_{\eta_S}^{\eta_O} d\eta\,\frac{(\eta-\eta_S)}{(\eta_O-\eta_S)}\dot{\Psi} + \int_{\eta_S}^{\eta_O} d\eta\,\frac{(\eta-\eta_S)(\eta_O-\eta)}{(\eta_O-\eta_S)}\ddot{\Psi}$$
$$- \int_{\eta_S}^{\eta_O} d\eta\,\frac{(\eta-\eta_S)(\eta_O-\eta)}{(\eta_O-\eta_S)}\nabla^2\Psi\Bigg\}. \qquad (59)$$

arXiv:2410.00956v1 [astro-ph.CO] 1 Oct 2024

Euclid preparation

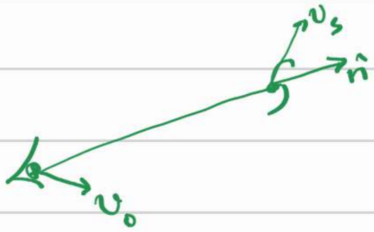
The impact of relativistic redshift-space distortions on two-point clustering statistics from the Euclid wide spectroscopic survey

Euclid Collaboration: M. Y. Elkhashab*[1,2], D. Bertacca[2,3,1], C. Porciani[4], J. Salvalaggio[5,6,7,8], N. Aghanim[9], A. Amara[10], S. Andreon[11], N. Auricchio[12], C. Baccigalupi[7,6,8,13], M. Baldi[14,12,15], S. Bardelli[12], C. Bodendorf[16], D. Bonino[17], E. Branchini[18,19,11], M. Brescia[20,21,22], J. Brinchmann[23], S. Camera[24,25,17], V. Capobianco[17], C. Carbone[26], V. F. Cardone[27,28], J. Carretero[29,30], R. Casas[31,32], S. Casas[33], M. Castellano[27], G. Castignani[12], S. Cavuoti[21,22], A. Cimatti[34], C. Colodro-Conde[35], G. Congedo[36], C. J. Conselice[37], L. Conversi[38,39], Y. Copin[40], F. Courbin[41], H. M. Courtois[42], A. Da Silva[43,44], H. Degaudenzi[45], A. M. Di Giorgio[46], J. Dinis[43,44], M. Douspis[9], F. Dubath[45], C. A. J. Duncan[37], X. Dupac[39], S. Dusini[1], M. Farina[46], S. Farrens[47], S. Ferriol[40], P. Fosalba[31,48], M. Frailis[6], E. Franceschi[12], S. Galeotta[6], B. Gillis[36], C. Giocoli[12,49], P. Gómez-Alvarez[50,39], A. Grazian[3], F. Grupp[16,51], L. Guzzo[52,11], S. V. H. Haugan[53], W. Holmes[54], F. Hormuth[55], A. Hornstrup[56,57], K. Jahnke[58], M. Jhabvala[59], B. Joachimi[60], E. Keihänen[61], S. Kermiche[62], A. Kiessling[54], M. Kilbinger[47], T. Kitching[63], B. Kubik[40], K. Kuijken[64], M. Kümmel[51], M. Kunz[65], H. Kurki-Suonio[66,67], S. Ligori[17], P. B. Lilje[53], V. Lindholm[66,67], I. Lloro[68], G. Mainetti[69], E. Maiorano[12], O. Mansutti[6], O. Marggraf[4], K. Markovic[54], N. Martinet[70], F. Marulli[71,12,15], R. Massey[72], E. Medinaceli[12], S. Mei[73], Y. Mellier[74,75], M. Meneghetti[12,15], G. Meylan[41], M. Moresco[71,12], L. Moscardini[71,12,15], S.-M. Niemi[76], C. Padilla[77], S. Paltani[45], F. Pasian[6], K. Pedersen[78], V. Pettorino[76], S. Pires[47], G. Polenta[79], M. Poncet[80], L. A. Popa[81], L. Pozzetti[12], F. Raison[16], R. Rebolo[35,82], A. Renzi[2,1], J. Rhodes[54], G. Riccio[21], E. Romelli[6], M. Roncarelli[12], R. Saglia[51,16], Z. Sakr[83,84,85], A. G. Sánchez[16], D. Sapone[86], M. Schirmer[58], P. Schneider[4], T. Schrabback[87], M. Scodeggio[26], A. Secroun[62], E. Sefusatti[6,7,8], G. Seidel[58], S. Serrano[31,88,32], C. Sirignano[2,1], G. Sirri[15], L. Stanco[1], J. Steinwagner[16], C. Surace[70], P. Tallada-Crespí[29,30], A. N. Taylor[36], I. Tereno[43,89], R. Toledo-Moreo[90], F. Torradeflot[30,29], I. Tutusaus[84], L. Valenziano[12,91], T. Vassallo[51,6], G. Verdoes Kleijn[92], A. Veropalumbo[11,19,93], Y. Wang[94], J. Weller[51,16], G. Zamorani[12], E. Zucca[12], A. Biviano[6,7], A. Boucaud[73], E. Bozzo[45], C. Burigana[95,91], M. Calabrese[96,26], D. Di Ferdinando[15], J. A. Escartin Vigo[16], R. Farinelli[12], F. Finelli[12,91], J. Gracia-Carpio[16], N. Mauri[34,15], A. Pezzotta[16], M. Pöntinen[66], V. Scottez[74,97], M. Tenti[15], M. Viel[7,6,13,8,98], M. Wiesmann[53], Y. Akrami[99,100], V. Allevato[21], S. Anselmi[1,2,101], A. Balaguera-Antolinez[35,82], M. Ballardini[102,12,103], A. Blanchard[84], L. Blot[104,101], H. Böhringer[16,105,106], S. Borgani[5,7,6,8], S. Bruton[107], R. Cabanac[84], A. Calabro[27], G. Canas-Herrera[76,108], A. Cappi[12,109], C. S. Carvalho[89], T. Castro[6,8,7,98], K. C. Chambers[110], A. R. Cooray[111], S. Davini[19], B. De Caro[26], S. de la Torre[70], G. Desprez[112], A. Díaz-Sánchez[113], J. J. Diaz[114], S. Di Domizio[18,19], H. Dole[9], S. Escoffier[62], A. G. Ferrari[34,15], P. G. Ferreira[115], I. Ferrero[53], A. Finoguenov[66], A. Fontana[27], F. Fornari[91], L. Gabarra[115], K. Ganga[73], J. García-Bellido[99], E. Gaztanaga[32,31,116], F. Giacomini[15], F. Gianotti[12], G. Gozaliasl[117,66], A. Hall[36], W. G. Hartley[45], H. Hildebrandt[118], J. Hjorth[119], A. Jimenez Muñoz[120], J. J. E. Kajava[121,122], V. Kansal[123,124], D. Karagiannis[125,126], C. C. Kirkpatrick[61], F. Lacasa[127,9,65], J. Le Graet[62], L. Legrand[128], A. Loureiro[129,130], G. Maggio[6], M. Magliocchetti[46], F. Mannucci[131], R. Maoli[132,27], C. J. A. P. Martins[133,23], S. Matthew[36], L. Maurin[9], R. B. Metcalf[71,12], M. Migliaccio[134,135], P. Monaco[5,6,8,7], C. Moretti[13,98,6,7,8], G. Morgante[12], S. Nadathur[116], Nicholas A. Walton[136], L. Patrizii[15], V. Popa[81], D. Potter[137], P. Reimberg[74], I. Risso[93], P.-F. Rocci[9], M. Sahlén[138], A. Schneider[137], M. Sereno[12,15], G. Sikkema[92], A. Silvestri[108], P. Simon[4], A. Spurio Mancini[139,63], K. Tanidis[115], C. Tao[62], N. Tessore[60], G. Testera[19], R. Teyssier[140], S. Toft[57,141,142], S. Tosi[18,19], A. Troja[2,1], M. Tucci[45], C. Valieri[15], J. Valiviita[66,67], D. Vergani[12], F. Vernizzi[143], G. Verza[144,145], P. Vielzeuf[62], and C. Hernández-Monteagudo[82,35]

(Affiliations can be found after the references)

October 3, 2024

Due to Two reasons, we obtain distortion in observation



(A) Z-Distortion

(B) Geometry Distortion

$$\bar{z} \to z , \qquad (1+z) = (1+\bar{z})\left(1 + \frac{\bar{v}_s \cdot \hat{n}}{c} - \frac{v_o \cdot \hat{n}}{c}\right)$$

$$d_A = \sqrt{\frac{\delta A_s}{\delta \Omega_o}}$$

$$d_A = \frac{S(\chi)}{1+z}$$

$$d_L = (1+z)^2 d_A$$

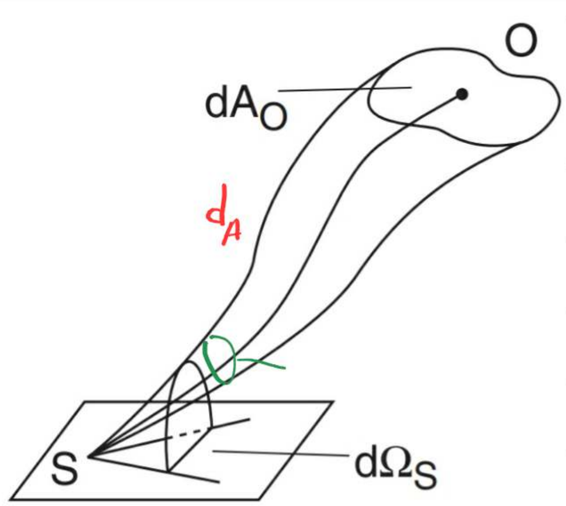


FIG. 1. A light beam emitted at the source event S ending on the observer O. At the source position, the plane normal to the source four-velocity is indicated.

after Perturbation

$$d_L \longrightarrow d_L = (1+\bar{z})^2 \left(1 + \frac{\bar{v}_s \cdot \hat{n}}{c} - \frac{v_o \cdot \hat{n}}{c}\right)^2 \sqrt{\frac{\delta A_s}{\delta \Omega_o}} \left(1 - \frac{\bar{v}_s \cdot \hat{n}}{c} + \frac{v_o \cdot \hat{n}}{c}\right)$$

$$ds^2 = a^2\left[c^2 d\eta^2 - d\chi^2 - S^2(\chi)\, d\Omega^2\right]$$

$$= a^2\left[c^2 d\eta^2 - \delta_{ij}\, dx^i dx^j\right]$$

$c=1$

$d\eta \equiv \frac{dt}{a(t)}$

Conformal Time

Euclid preparation

The impact of relativistic redshift-space distortions on two-point clustering statistics from the Euclid wide spectroscopic survey

$$ds^2 = c^2 dt^2 - a^2(t)\left[d\chi^2 + S^2(\chi)\, d\Omega^2\right]$$ FRW_metric

همگن و همسانگرد

$$\mathrm{d}s^2 = a^2(\eta)\left[-(1 + 2\Psi)\, c^2\, \mathrm{d}\eta^2 + (1 - 2\Phi)\, \delta_{\mathrm{K}\,ij}\, \mathrm{d}x^i\, \mathrm{d}x^j\right], \tag{3}$$

where Ψ and Φ are the dimensionless Bardeen potentials, η is the conformal time, and a is the cosmic scale factor. From this choice, it follows that (Hui & Greene 2006, Yoo et al. 2009, Bonvin & Durrer 2011, Challinor & Lewis 2011, Jeong et al. 2012)

$$\Delta x^0 = \frac{c}{\mathcal{H}}\, \delta \ln a\,, \tag{4a}$$

$$\begin{aligned}\Delta x^i = &-\left(\Phi_{\mathrm{o}} + \Psi_{\mathrm{o}} + \frac{\boldsymbol{v}_{\mathrm{e}}}{c} \cdot \boldsymbol{n}\right) x^i - x\, \frac{v^i_{\mathrm{o}}}{c} - \frac{c}{\mathcal{H}}\, n^i\, \delta \ln a \\ &+ n^i \int_0^x (x - \tilde{x})\, \frac{(\Phi' + \Psi')}{c}\, \mathrm{d}\tilde{x} - \int_0^x (x - \tilde{x})\, \delta^{ij}_{\mathrm{K}}\, \tilde{\partial}_j(\Phi + \Psi)\, \mathrm{d}\tilde{x} \\ &+ 2n^i \int_0^x (\Phi + \Psi)\, \mathrm{d}\tilde{x}\,,\end{aligned} \tag{4b}$$

Red Shift

where

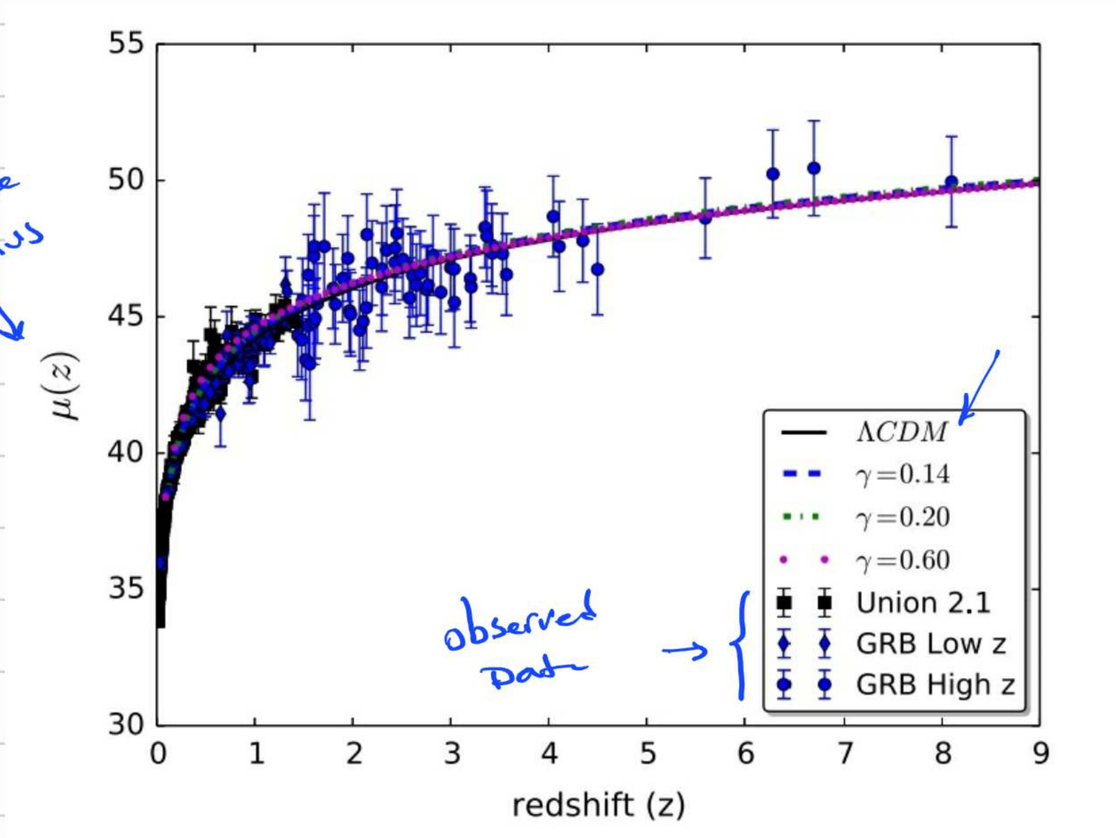
Gravitational Time-Dependancy of potential

$$\delta \ln a := \left[\underbrace{\frac{(\boldsymbol{v}_{\mathrm{e}} - \boldsymbol{v}_{\mathrm{o}})}{c}}_{\text{سرعت خاصه}} \cdot \boldsymbol{n} - \underbrace{(\Phi_{\mathrm{e}} - \Phi_{\mathrm{o}}) - \int_0^x \frac{(\Phi' + \Psi')}{c}\, \mathrm{d}\tilde{x}}_{\text{اختلالات مشترک}}\right], \tag{5}$$

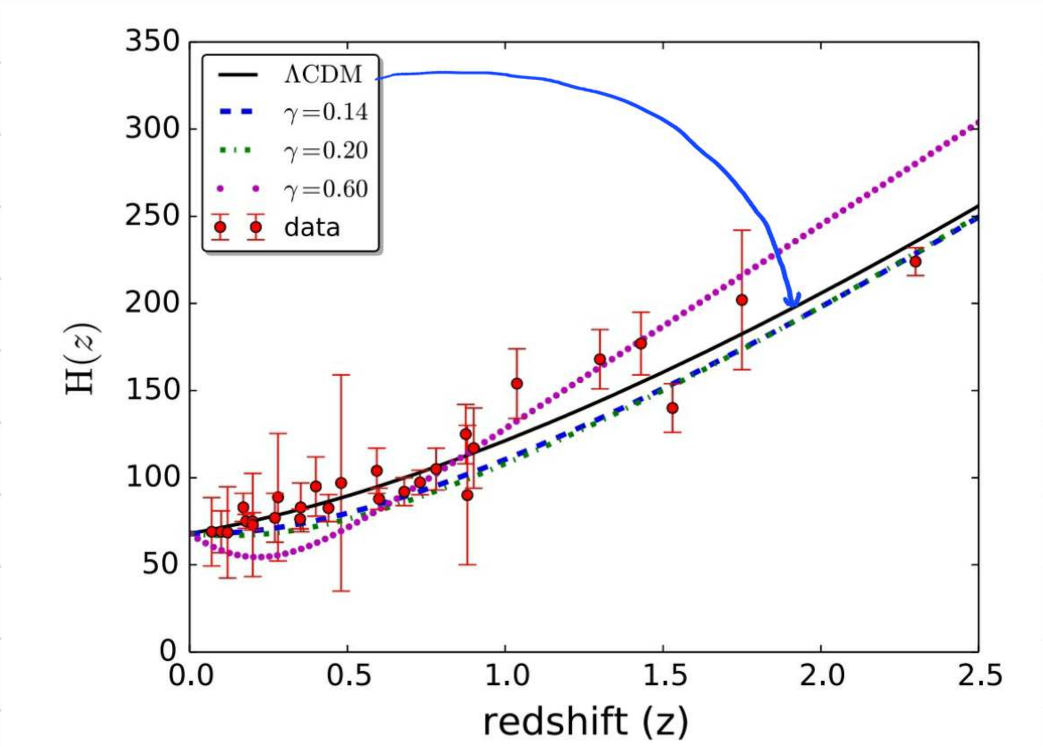
Spatial part of Perturbation

arxiv: 2410.00956

Distance Modulus

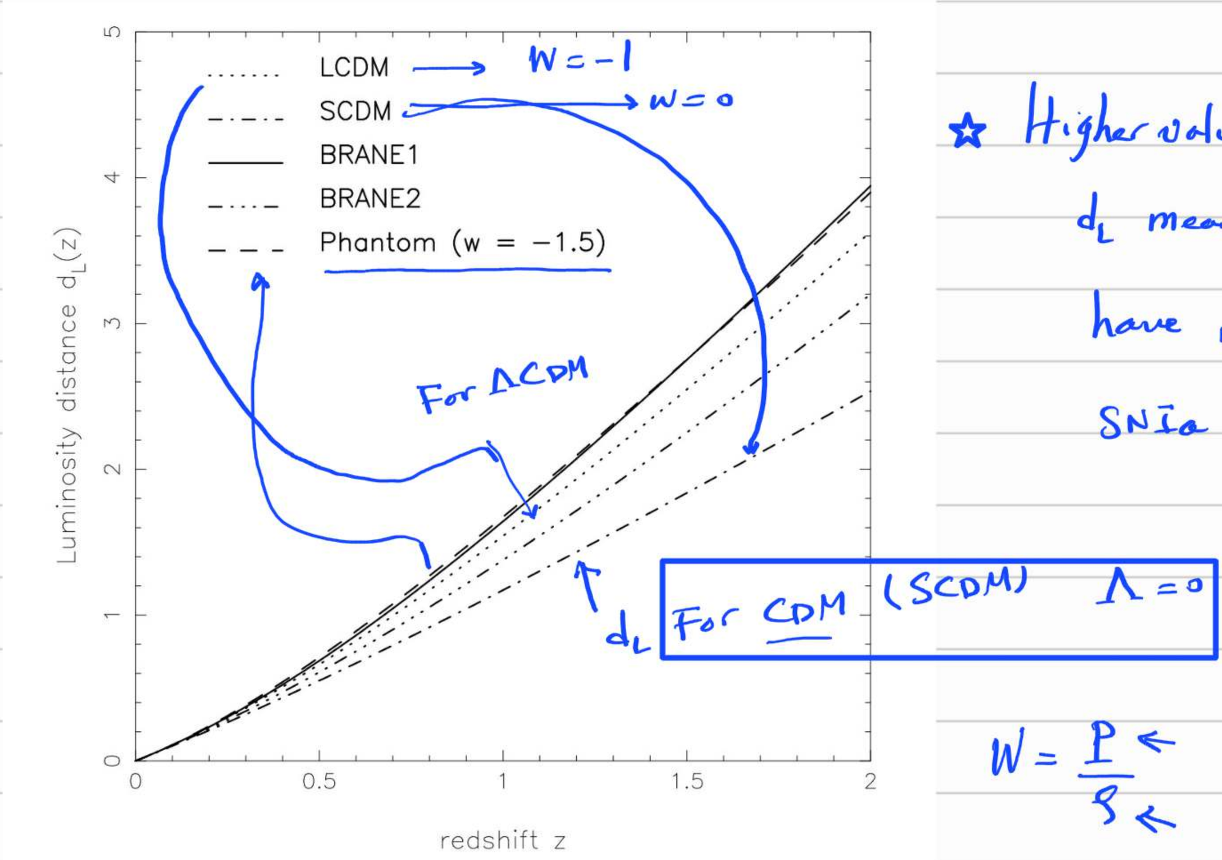


Observed Data Set



We need to have lower value of H at low Redshift Compared to CDM model.

$$d_L = \frac{(1+z)c}{H_0}\int_0^z \frac{dz'}{\underline{H(z')}}$$



☆ Higher value of d_L means to have more faint SNIa

$W = \frac{P}{\rho}$ ← فشار ← چگالی

$Z \in [0, 2]$ low Redshift interval

we want to have higher d_L

Braneworld models of dark energy

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arXiv:astro-ph/0202346v3 9 Dec 2003

Abstract

We explore a new class of braneworld models in which the scalar curvature of the (induced) brane metric contributes to the brane action. The scalar curvature term arises generically on account of one-loop effects induced by matter fields residing on the brane. Spatially flat braneworld models can enter into a regime of accelerated expansion at late times. This is true even if the brane tension and the bulk cosmological constant are tuned to satisfy the Randall–Sundrum constraint on the brane. Braneworld models admit a wider range of possibilities for dark energy than standard LCDM. In these models the luminosity distance can be both smaller and larger than the luminosity distance in LCDM. Whereas models with $d_L \leq d_L$(LCDM) imply $w = p/\rho \geq -1$ and have frequently been discussed in the literature, mod-

Ex4: The Compact Form of d_L.

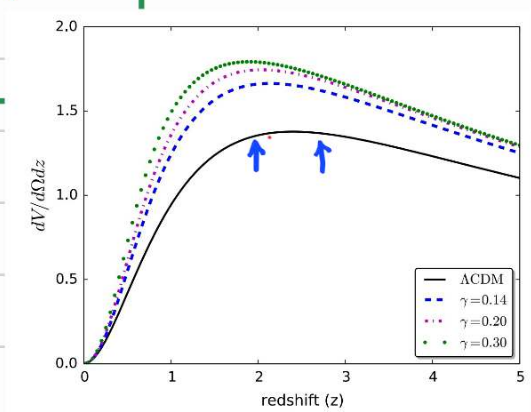
$$d_L = \frac{(1+z)c}{H_0} \frac{1}{\sqrt{|k|}} \operatorname{Sinn}\left[\sqrt{|k|}\int_0^z \frac{dz'}{H(z')}\right]$$

$$d_L = \underbrace{\frac{c}{H_0}}\, d_L' = \frac{3000\,\mathrm{Mpc}}{h}\, d_L' \rightarrow \text{Dimension-Less}$$

$$d_L' \equiv \frac{(1+z)}{\sqrt{|k|}} \operatorname{Sinn}\left[\sqrt{|k|}\int_0^z \frac{dz'}{H(z')}\right]$$

$$\mu = 5\log d_L + 25$$

$$\boxed{\mu = 5\log(d_L') + 5\log\left(\frac{c/H_0}{\mathrm{Mpc}}\right) + 25}$$



(5) Comoving Volume Element

$$CV \equiv \frac{dV}{d\Omega\, dz} = \frac{d\Omega\, S^2(\chi)\, d\chi}{d\Omega\, dz} = S^2(\chi)\frac{d\chi}{dz}$$

$$d\chi = c\frac{dz}{H(z)}$$

$$\boxed{CV = \frac{S^2(\chi)\, c}{H(z)}}$$

Ex5: How many source are available at z and $z+dz$?

$$\overset{N(z,\hat{n})}{\frac{dN}{dz}} = n(z)\frac{dV}{dz} = n(z)\, 4\pi\, CV$$

Ex6: How many galaxy exists such that $\underline{L} > L_{min}$

☆ $dN = \frac{dN}{dL\,dz}\, dL\, dz$ ☆

$$N = \int_{L_{min}}^{\infty} dN = \int dz\, d_L\; n(L,z)\, 4\pi \frac{S^2(x)}{H(z)} \frac{1}{(1+z)^3}$$

☆ Also see Sec 1.11 of Cosmology written by S. Weinberg ☆
[Number counts] Page 83, 84

A practical theorem on gravitational wave backgrounds

E.S. Phinney*
Theoretical Astrophysics, 130-33 Caltech, Pasadena, CA 91125, USA

2001 July 31

Application for GW sources production

Now consider sources undergoing the catastrophic events at redshift z, at rate N per comoving volume per unit of cosmic time t_r local to the event. As seen from earth, in earth time dt, the number of events which occur in dt between redshift z and $z + dz$ is

$$\frac{d\#}{dt\,dz} = \dot{N} \frac{1}{1+z} \frac{d\mathcal{V}_c}{dz}, \quad \longrightarrow \frac{d^2N}{dt\,dz} \qquad (27)$$

$\leftarrow$ Abundance of source making in z and dz

where the comoving volume element is (cf. Hogg (2000) and references therein)

$$\frac{d\mathcal{V}_c}{dz} = 4\pi \frac{c}{H_0} d_M^2 \frac{1}{E(z)}, \qquad (28)$$

where $E(z)$ was defined in equation 14. The number of events which occur in a comoving volume between the cosmic times $t_r(z)$ and $t_r(z + dz)$ is

$$N(z) = \dot{N} \frac{dt_r}{dz} = \dot{N} \frac{1}{(1+z)H_0 E(z)}. \qquad (29)$$

Thus equation 27 can be rewritten

$$\frac{d\#}{dt\,dz} = N(z) c 4\pi d_M^2. \qquad (30)$$

$$ds^2 = c^2 dt^2 - a^2(t)\left[d\chi^2 + S(\chi)^2 d\Omega^2\right]$$

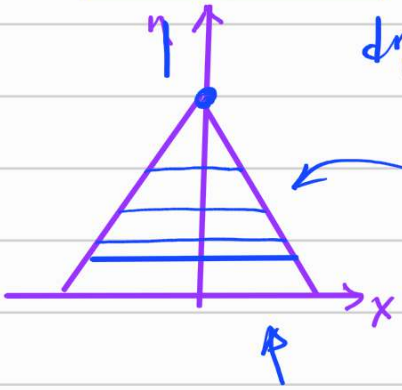
$$= a^2\left[d\eta^2 - d\chi^2 - S^2(\chi)\, d\Omega^2\right]$$

$$\boxed{d\chi = \frac{dt}{a(t)}}$$

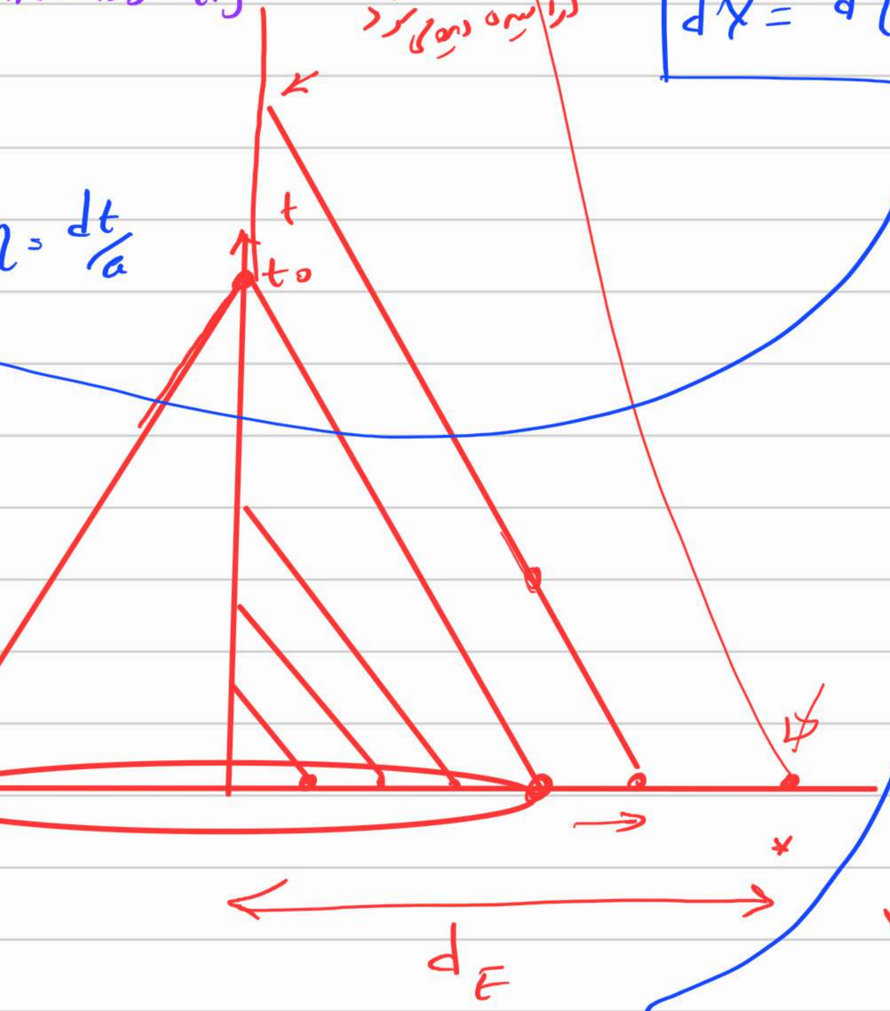
For Radial propagation of light

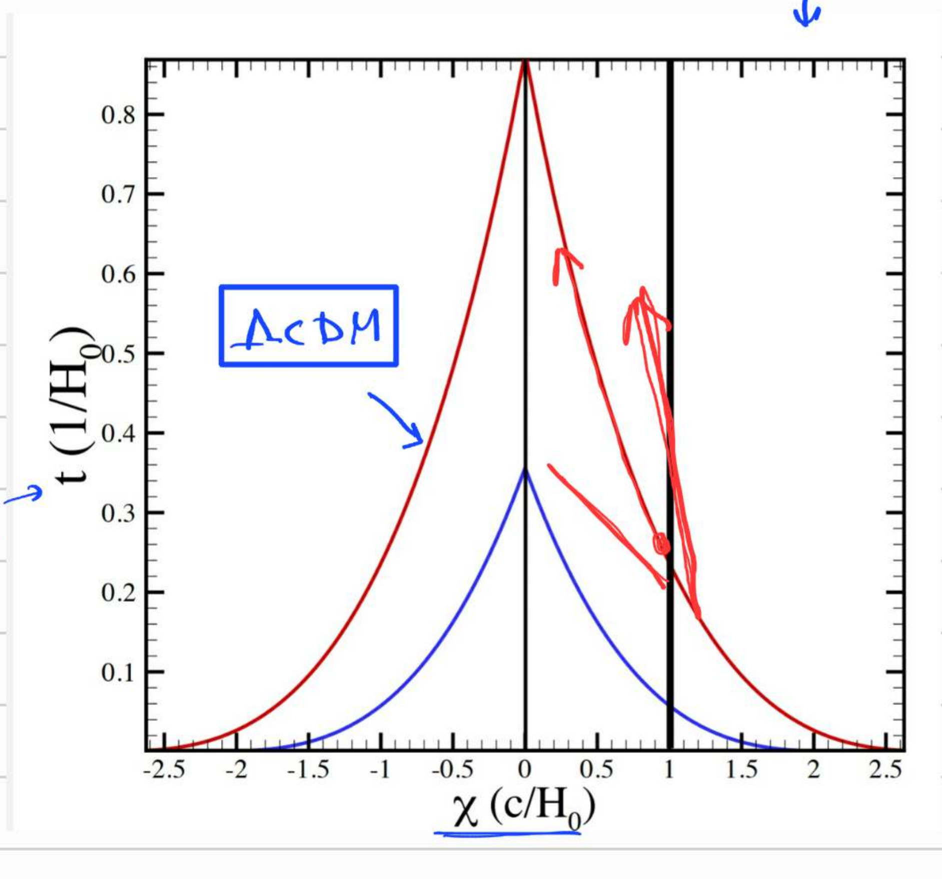
$$\boxed{d\chi = d\eta}$$

$$\boxed{\chi(\eta) = \pm \eta}$$



$$d\eta = \frac{dt}{a}$$





① $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho(t)$ $\leftarrow$ معادله اول فریدمان

② $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$ $\leftarrow$ معادله دوم

$a(t) = \checkmark$