Name:

In the name of God

Department of Physics
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ADVANCED COMPUTATIONAL PHYSICS

Final exam

(Time allowed: 3 hours)

NOTE: All question must be answered. Legibility, good hand-writing and penmanship have 5 additional marks. Please write the answer of each question in separate sheet. Upon you finish the theoretical part and giving back your answer sheet, you are allowed to solve the computational part.

Theoretical part

1. Transformation of variables: suppose that we have stochastic variables in D-Dimension such as $\{x\}$: $x_1, x_2, ..., x_D$ and its multivariate probability distribution function is given by $p(\{x\})$, assume that there is a mapping as: $\{x\} \rightarrow \{y\}$,

   (a) If the shape of $p(\{x\})$ is given calculate $p(\{y\})$. (10 marks).

   (b) Write down the detail of so-called Box-Muller method to make an artificial Gaussian random series. (10 marks).

2. Fisher information matrix. (10 marks)

   Suppose the likelihood is given by: $\mathcal{L} = \frac{1}{\sqrt{(2\pi)^N Det(\mathbf{C})}} \exp(-\Delta^T.\mathbf{C}^{-1}.\Delta/2)$, where $\Delta \equiv \mathbf{X}(\{\Theta\}) - d$ and

   $\mathbf{C}$ is the covariant matrix of observation. If $\mathcal{F}_{ij} \equiv -\langle \frac{\partial^2 \ln \mathcal{L}}{\partial \theta_i \partial \theta_j} \rangle$, show that $\vec{r}$:

$$\mathcal{F}_{ij} = \frac{\partial \mathbf{X}^T}{\partial \theta_i} \mathbf{C}^{-1} \frac{\partial \mathbf{X}}{\partial \theta_j} + \frac{1}{2} \mathrm{Tr} \left(\mathbf{C}^{-1} \frac{\partial \mathbf{C}}{\partial \theta_i} \mathbf{C}^{-1} \frac{\partial \mathbf{C}}{\partial \theta_j} \right) \tag{1}$$

3. Energy of 2-D Ising model on the square lattice. (20 marks)

   Suppose that the hamiltonian of 2-D Ising model is as follows: $\mathcal{H} = -J \sum_{\langle ij \rangle} S_i S_j - B_{\text{ext}} \sum_i S_i$. Explain in detail the subroutine for calculation of changing the energy when an arbitrary spin is selected in a Monte Carlo step and to be reversed.

Computational part

4. Lorentz attractor(1963)(20 marks):

Solve the following coupled differential equation numerically and plot $x_3(t)$ as a function of $x_1(t)$ and $x_2(t)$ to show the chaotic phase.

$$\frac{dx_1(t)}{dt} = \xi_1(x_2(t) - x_1(t))$$
$$\frac{dx_2(t)}{dt} = \xi_3 x_1(t) - x_2(t) - x_1(t)x_3(t)$$
$$\frac{dx_3(t)}{dt} = x_1(t)x_2(t) - \xi_2 x_3(t)$$

with following initial conditions:

$$x_1(0) = 1, x_1'(0) = 1, x_2(0) = 1, x_2'(0) = 0, x_3(0) = 1, x_3'(0) = 1, \xi_1 = 10, \xi_2 = 8/3, \xi_3 = 28 \qquad (2)$$

Good luck, Movahed