بسم الله الرحمن الرحیم

outline

※ More about Hurst Exponent

※ Data Modeling.

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① $y(ax) \triangleq a^{\xi} y(x)$ — Self-Similarity of Data.

$\xi \equiv$ Hurst Exponent.
$= H$
→ Clarification of so-called (جهان شمولی) Universality

Most Relevant Exponents $\left\{ \gamma, \beta, \tau(q), D(q), \vdots \right\}$

$D(fractal-dimention)$	H_{fBm}	H_{fGn}	β	$h(q=۲)$	
$-$	$۲-H_{fBm}$	$-$	$\frac{۵-\beta}{۲}$	$۳-h(q=۲)$	D
$۲-D$	$-$	$-$	$\frac{\beta-۱}{۲}$	$h(q=۲)-۱$	H_{fBm}
$-$	$-$	$-$	$\frac{\beta+۱}{۲}$	$h(q=۲)$	H_{fGn}
$۵-۲D$	$۲H_{fBm}+۱$	$۲H_{fGn}-۱$	$-$	$۲h(q=۲)-۱$	β
$۳-D$	$H_{fBm}+۱$	H_{fGn}	$\frac{\beta+۱}{۲}$	$-$	$h(q=۲)$

q	$\tau(q)$	$\alpha = -\frac{d\tau(q)}{dq}$	$f = q\alpha + \tau(q)$
$q \to -\infty$	$-q\alpha_{max}$	$\alpha_{max} = -\ln \mu_{-} / \ln \delta$	0
$q = 0$	D	α_0	D
$q = 1$	0	$\alpha_1 = -S(\delta)/\ln \delta$	α_1
$q \to +\infty$	$-q\alpha_{min}$	$\alpha_{min} = -\ln \mu_{+} / \ln \delta$	0

Fractals: Jens Feder 1988

② Questions

☆ Self-Similar (Self-affine) or no ?

☆ How to quantify? ⟶ $\xi = ?$

☆ ξ ⟶ Hurst Exponent ?

MF-DFA algorithm

Multi-Fractal Detrended Fluctuation in 1D

$Y(at) \stackrel{?}{=} a^{H} Y(t)$ — $H = ?$

DFAm remove trend of order m in profile or trend of order m-1 in original seris

Step 0: Import Data $\{x\}_{,i=1,N}$ ⟶ Z-Transform

- *Step 1.* Determine the 'profile'

$$Y(i) \equiv \sum_{k=1}^{i} [x_k - \langle x \rangle], \qquad i = 1, \ldots, N.$$

$Y(t) = \int_0^t dt' x(t')$

loop $N_s = 4, \ldots N/4$

- *Step 2.* Divide the profile $Y(i)$ into $N_s \equiv \text{int}(N/s)$ non-overlapping segments of equal lengths s.

$s = \text{int}(N/N_s)$

- *Step 3.* Calculate the local trend for each of the $2N_s$ segments by a least squares fit of the series. Then determine the variance

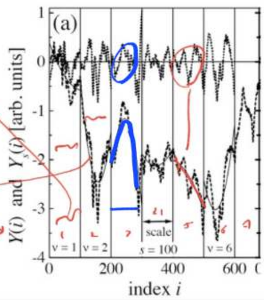
$$F^2(s, \nu) \equiv \frac{1}{s} \sum_{i=1}^{s} \{Y[(\nu - 1)s + i] - y_\nu(i)\}^2$$

for each segment ν, $\nu = 1, \ldots, N_s$ and

$$F^2(s, \nu) \equiv \frac{1}{s} \sum_{i=1}^{s} \{Y[N - (\nu - N_s)s + i] - y_\nu(i)\}^2,$$

for $\nu = N_s + 1, \ldots, 2N_s$.

Here, $y_\nu(i)$ is the fitting polynomial in segment ν. Stop



- *Step 4.* Average over all segments to obtain the qth-order fluctuation function, defined as

$$F_q(s) \equiv \left\{ \frac{1}{2N_s} \sum_{\nu=1}^{2N_s} [F^2(s, \nu)]^{q/2} \right\}^{1/q}$$

$F_q(s)$ is only defined for $s \geq m + 2$.

- *Step 5.* Determine the scaling behaviour of the fluctuation functions by analysing log–log plots of $F_q(s)$ versus s for each value of q. If the series x_i are long range power law correlated, $F_q(s)$ increases, for large values of s, as a power law,

$$F_q(s) \sim s^{h(q)}.$$

* Demand *

$h(q) \to \xi$

$\xi \in \mathbb{R}$ ↑ Multi-fractality

$\{X\} : \{F_q(s)\} \qquad \{\Theta\} : \{h(q)\}$

$$P(h(q)|X) = \frac{\mathcal{L}(X|h(q))P(h(q))}{\int \mathcal{L}(X|h(q))dh(q)} \qquad \mathcal{L}(X|h(q)) \sim \exp\left(\frac{-\chi^2(h(q))}{2}\right)$$

$$\chi^2(h(q)) = \int ds \frac{[F_{\text{obs.}}(s) - F_{\text{The.}}(s; h(q))]^2}{\sigma^2_{\text{obs.}}(s)}$$

$$68.3\% = \int_{-\sigma^-}^{+\sigma^+} \mathcal{L}(X|h(q))dh(q) \qquad h^{+\sigma^+}_{-\sigma^-}$$

Jan W. Kantelhart, et. al., arXiv:physics/0202070; M. Sadegh Movahed et. al., arXiv:physics/0508149
S. Hajian and M. Sadegh Movahed, arXiv:0908.0132

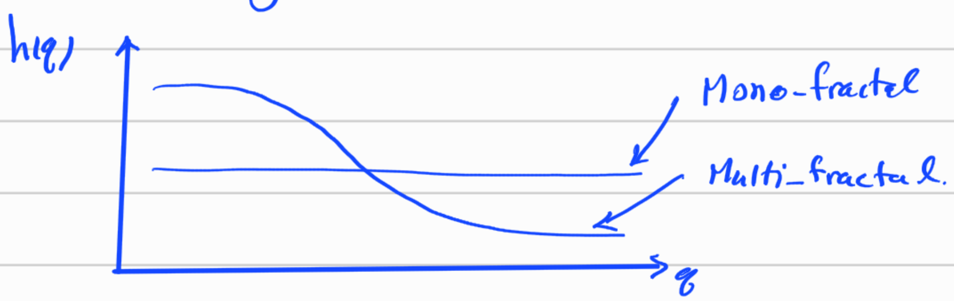
fGn $\quad h(q=2) = H$

fBm $\quad h(q=2) = H + 1$

$0 < h(q=2) < 1 \longrightarrow$ Stationary $\leftarrow$ fGn

$1 < h(q=2) \longrightarrow$ Non_Stationary $\leftarrow$ fBm

q-Dependency indicates Multi-fractality.



③ Surrogate & shuffled. : Source of Multi-fractality

Surrogate: بُر زدن فاز (در فضای فوریه)
$\downarrow$
$P(y) \longrightarrow \mathcal{N}(\bar{y}, \sigma_y)$
$\nearrow$ Input Data

shuffled: بُر زدن $\longleftarrow$ $C_y(\tau) \longrightarrow C_y(\tau) = \delta_D(\tau)$

هرگونه اطلاعات موجود در تابع همبستگی را نابود می‌کنند

هرگونه اطلاعات منتج در تابع توزیع به علت حالت گوسی منحرف شده را نابود می‌کند.

$\longrightarrow$ Sources of Multi-fractality.

$\longrightarrow$ ① Correlation

→ ② PDF

بُر زده شده

$$F_q(s)/F_q^{\text{shuf}}(s) \sim s^{h(q)-h_{\text{shuf}}(q)} = s^{h_{\text{cor}}(q)},$$

$$F_q(s)/F_q^{\text{sur}}(s) \sim s^{h(q)-h_{\text{sur}}(q)} = s^{h_{\text{PDF}}(q)}.$$

سوری شده

$h_{cor} \equiv h - h_{shuf}$

$h_{PDF} \equiv h - h_{sur}$

$h_{\text{cor}}(q) = 0$ For Fatness → $h(q) = h_{shuf}(q)$ →

$h_{\text{PDF}}(q) = 0$ For correlation → $h(q) = h_{sur}(q)$

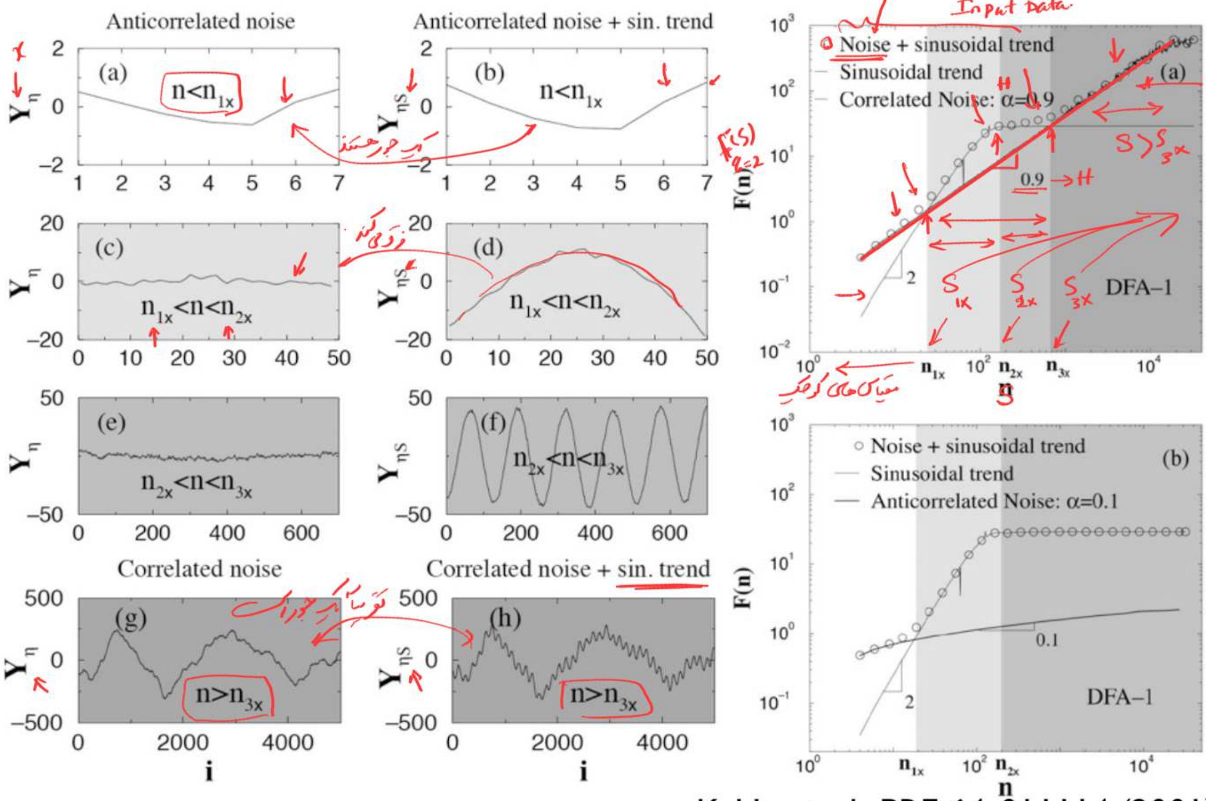
What are shuf" and "sur"?
Actually there are the abbreviation of
Shuffled and Surrogate data set

④ Imprint of Trend on Fractality or multi-fractality.

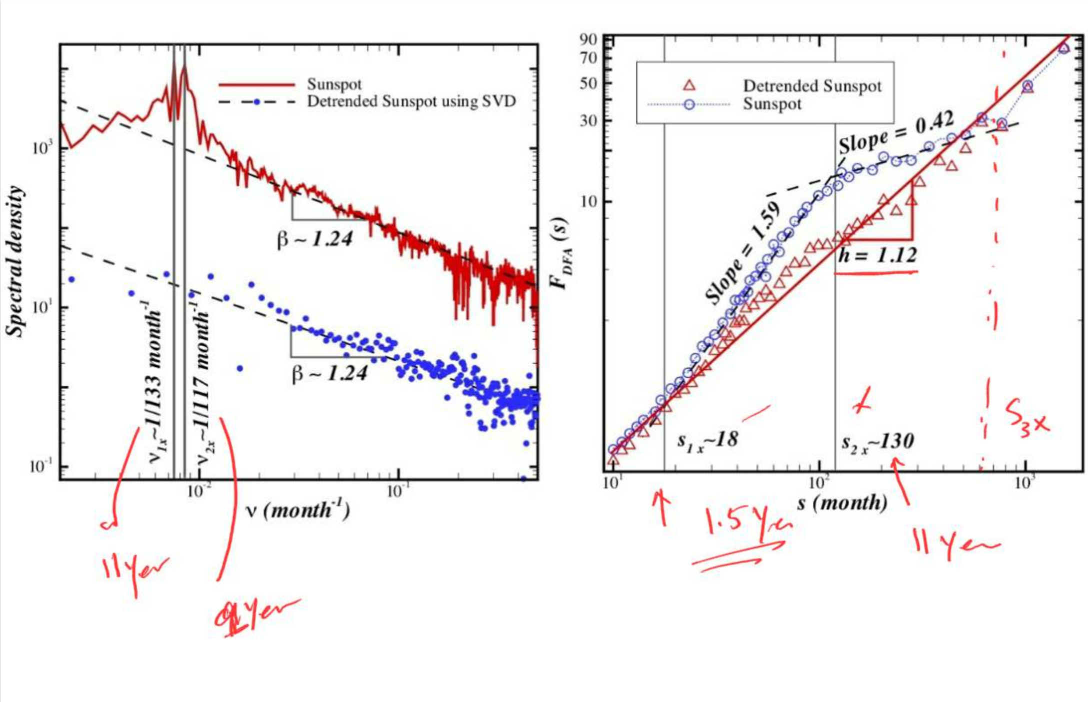
DFA: Detrended fluctuation Analysis.

Sinusoidal Trends

$x(t)$ — $x'(t) = x(t) + A\sin(\omega t)$



K. Hu et. al., PRE 64, 011114 (2001)



S. hajian and M. Sadegh Movahed, arXiv:0908.0132

Input Data

Sunspot

q=2

h=1.12 → H=0.12

h>1 → Non_Stat

⑤ Comments on Different Algorithms

DMA (BDMA & CDMA) and MF-TWDFA

Irregular DMA ← arXiv: 1704.08599

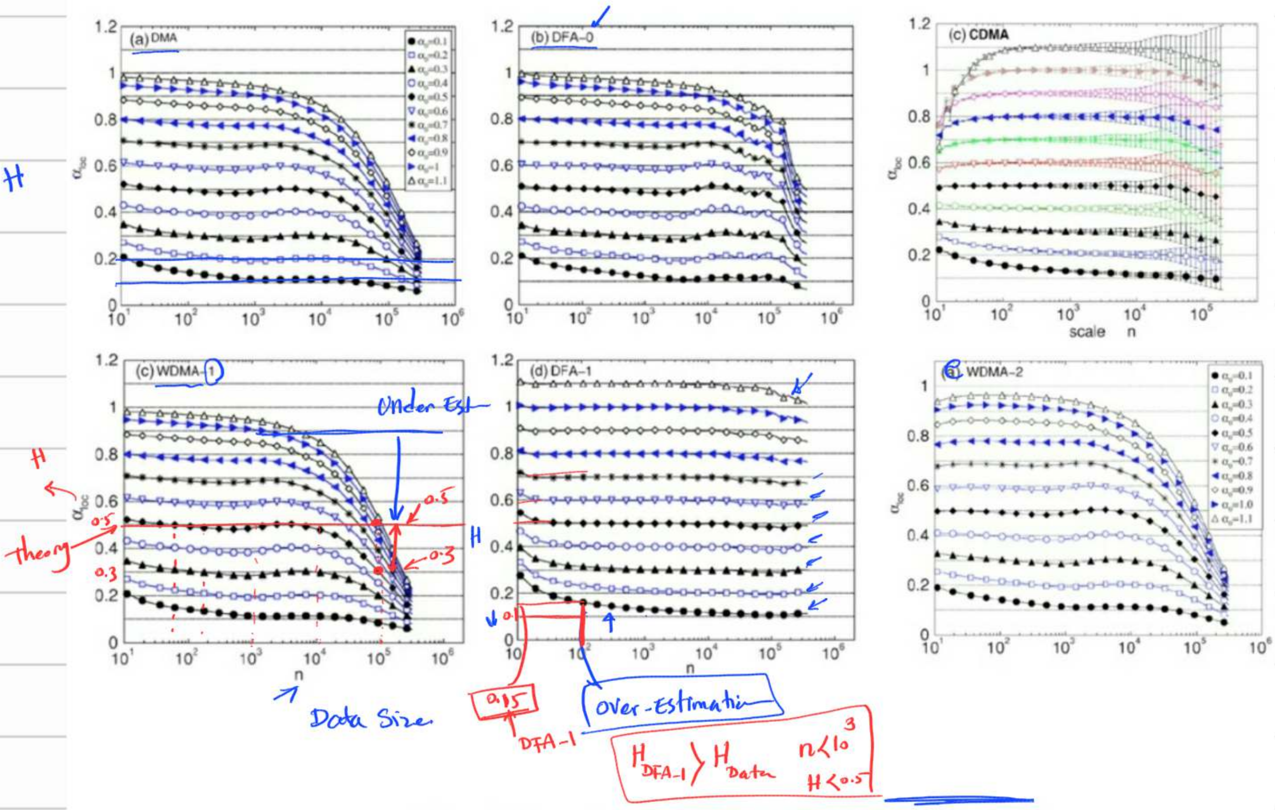
Refer to :

1) arXiv:cond-mat/0507395
2) PRE 71, 051101 (2005)
3) PRE 73, 016117 (2006)
4) JSTAT P06021 (2010)

☆ Accuracy

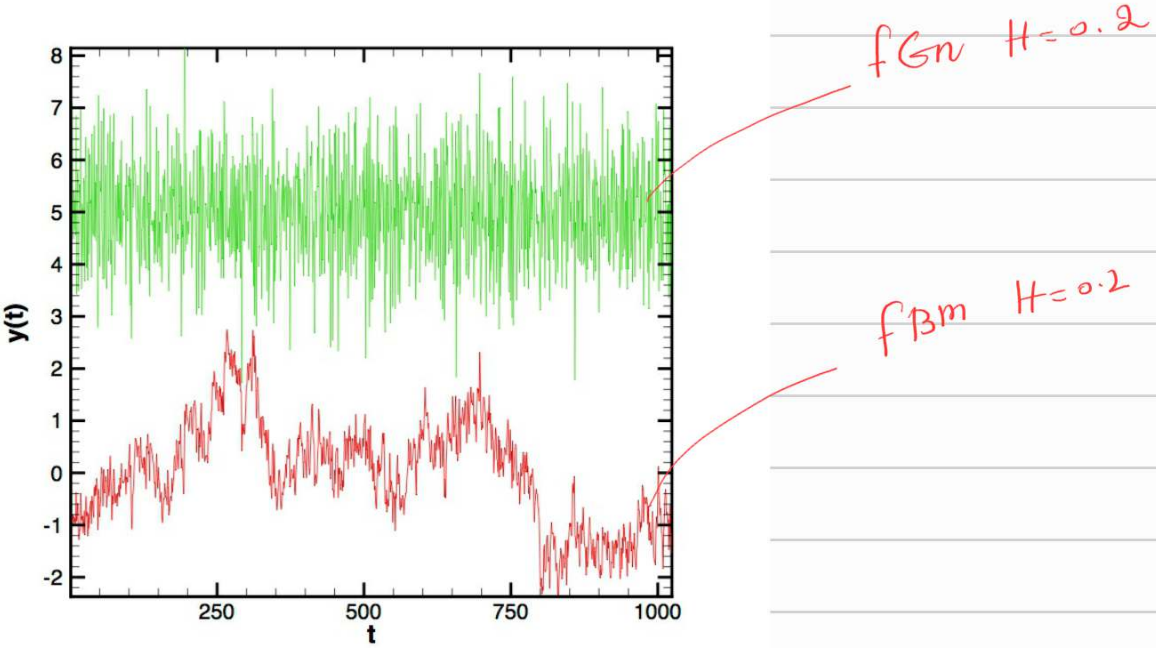
☆ Size-Dependency

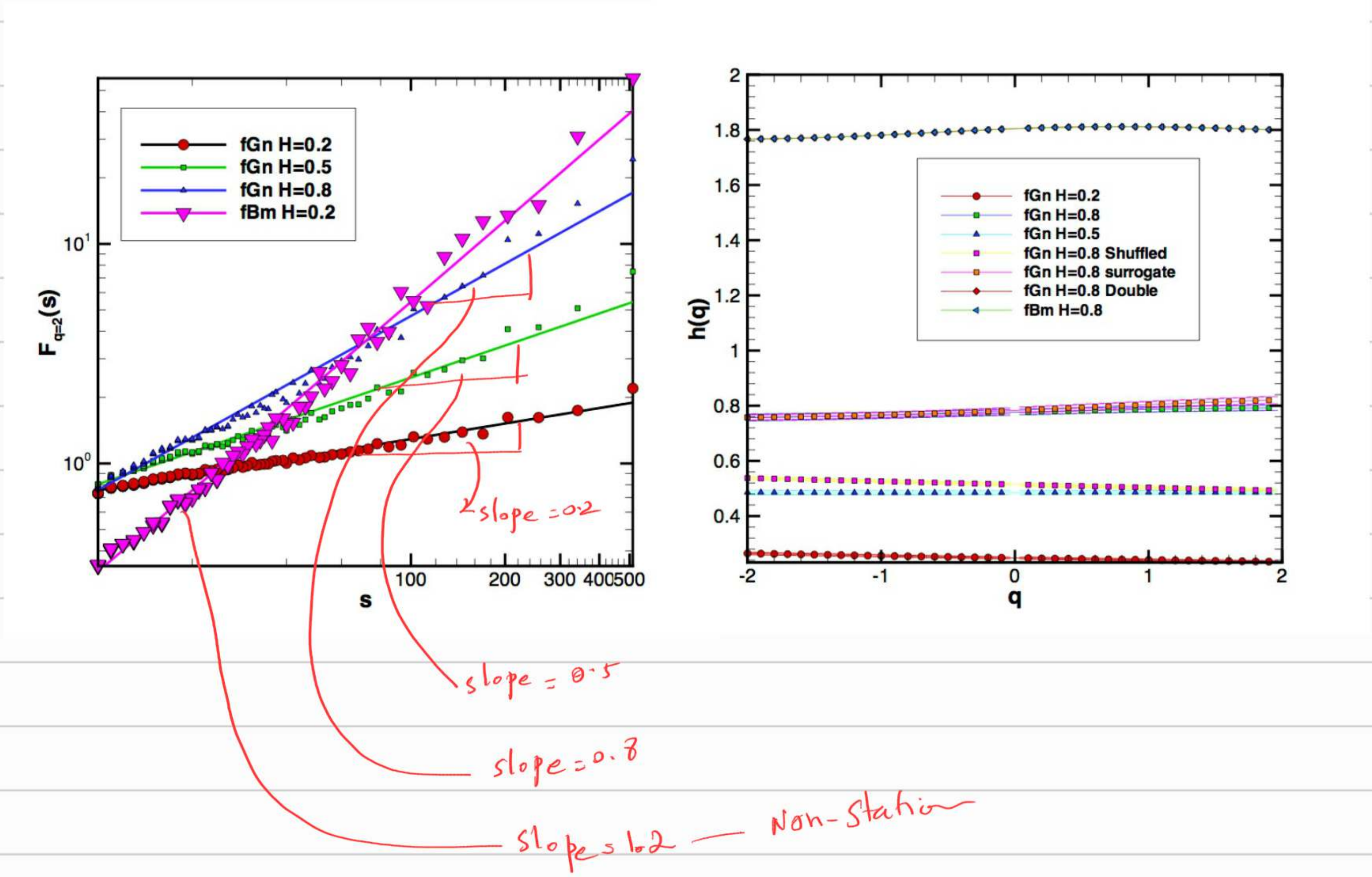
☆ Robustness with respect to trends.



Limei Xu et. al., PRE 71, 051101 (2005)

⑥ Examples. fBm & fGn





fGn H=0.2
fGn H=0.5
fGn H=0.8
fBm H=0.2
10^1
10^0
$F_{q=2}(s)$
s
100
200
300
400
500
slope = 0.2
slope = 0.5
slope = 0.8
slope = 1.2 — Non-Station
fGn H=0.2
fGn H=0.8
fGn H=0.5
fGn H=0.8 Shuffled
fGn H=0.8 surrogate
fGn H=0.8 Double
fBm H=0.8
h(q)
q
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
-2
-1
0
1
2