بسم الله الرحمن الرحيم

Summary on Data generator.

① Synthetic Data for a given Power spectrum

with Gaussian Distribution

(1+1)-Dimen $\{x_i\}$, $i=1, \dots N$ — $C_x(\tau) = \langle x(t+\tau)\, x(t) \rangle_\tau$

$$S_x(\omega) = \int d\tau\, e^{-i\omega\tau}\, C_x(\tau)$$

$$C_x(\tau) = \int d\omega\, e^{+i\omega\tau}\, S_x(\omega)$$

$$P(x) = \mathcal{N}(0, \sigma)$$

$$\sigma^2 = \int d\omega\, S_x(\omega)$$

For

(1+D)-Dimension

$\alpha(r^D)$

$$C_\alpha(R) = \langle \alpha(\vec{r}+\vec{R})\, \alpha(\vec{r}) \rangle_r$$

$$= \int d^d K\, e^{i\vec{K}\cdot\vec{R}}\, \underbrace{S_\alpha(K)}_{\displaystyle S_\alpha(|K|)}$$

$R = |\vec{r}+\vec{R}-\vec{r}|$

(Homogeniety)

$$\vec{K} = (K_1, K_2 \dots K_D)$$

Ex $\quad C(\tau) = e^{-\tau/\tau_c}$ ✓ $\longrightarrow$ $\{x(t)\}$, ✓

$\downarrow$

$\langle x(t+\tau)\,x(t)\rangle_t$

To be consistent

سازگار باشد

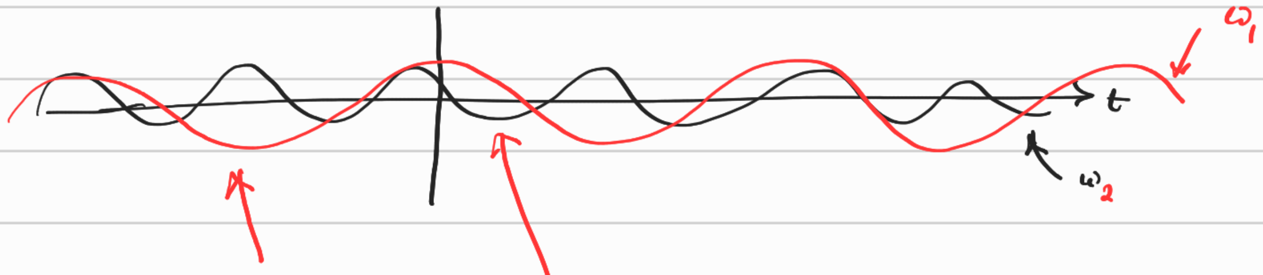
$P(x) = N \leftarrow$ Normal Distribu

② Wavelet (Mother wavelets, Curvelet, Ridgelet, . . .)

(امواج کوچک) موجک — مولدهای مرکب — خمک

$$\{x(t)\} \longrightarrow \underbrace{X(\omega)}_{\text{دامنه موج با فرکانس } \omega} = \int dt\, e^{-i\omega t}\, x(t)$$

گستردگی بی‌نهایت است. — $e^{-i\omega t}$ ($\cos(\omega t)$ or $\sin(\omega t)$)



$\cos(\omega_1 t)$ $\quad$ $\cos(\omega_2 t)$ — $(-\infty, +\infty)$

F.T $\longrightarrow$ Wavelet Transformation

$e^{-i\omega t} \longrightarrow$ Localized form

$$\begin{cases} x(t) = \int d\omega\, e^{+i\omega t} X(\omega) \\ X(\omega) = \int_{-\infty}^{+\infty} dt\, e^{-i\omega t} x(t) \end{cases}$$

$$\cancel{X}(s, a) = \int dt\ \underbrace{\psi(s, t-a)}\ x(t)$$

Base-Function

S≡Scale

$\omega_s \equiv \frac{1}{s}$

a=shift

$[a]=[t]$

$\cancel{\alpha},$

$\underbrace{s^{-1} \to \omega}$

$\psi(s^{-1}, t-a)$

plays the role of frequency

$[a]=[t] \to$ shift

$-\infty < a < +\infty$

$t_{min} \leq a \leq t_{max}$

$$\psi \to \bar{\psi}(s^{-1}, t) = \bar{\psi}(\omega, t) \equiv e^{-i\omega t}$$

become a simple F.T.

shift-Parameter

$$\psi - \underbrace{W\left(\frac{x-a}{s}\right)}$$

scale

What types of Wavelete have been considered?

③ Principle Component Analysis. (PCA)

geeksforgeeks.org

تحلیل مؤلفه های اصلی

Definition :

تبدیل متعامد

It is an Orthogonal transformation

$\vec{A}:(A_x, A_y, A_z)$

$A_x = \vec{A}\cdot\hat{i}$

$A_y = A\cdot\hat{j}$

$A_z = A\cdot\hat{k}$

$a(10^{-1})$ → $\left\{\begin{array}{l} 10 \\ 0.1 \end{array}\right.$

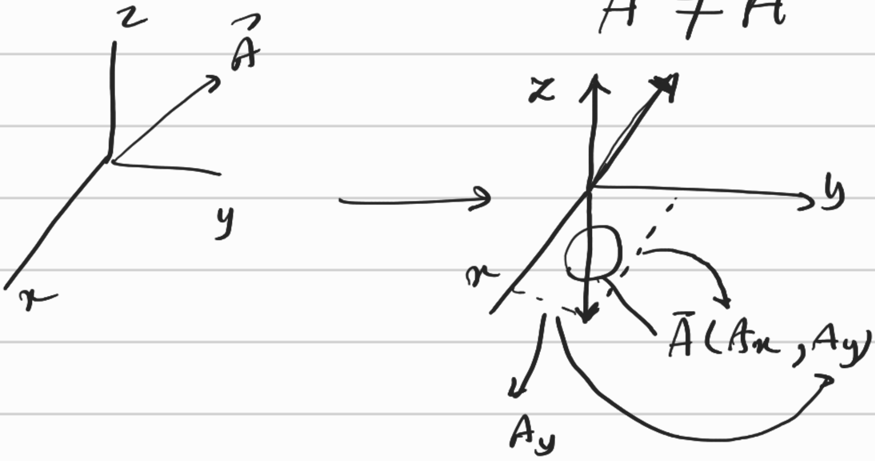
$a(10^{-3})$ → $\cancel{0.01}$

$i\cdot j = 0$

$\vec{A}(A_x, A_y, A_z) \longrightarrow \bar{A}(A_x, A_y)$ $\quad (A_x, A_y)$

{Dimensional Reduction}

$\bar{A} \neq A$ (information loss)

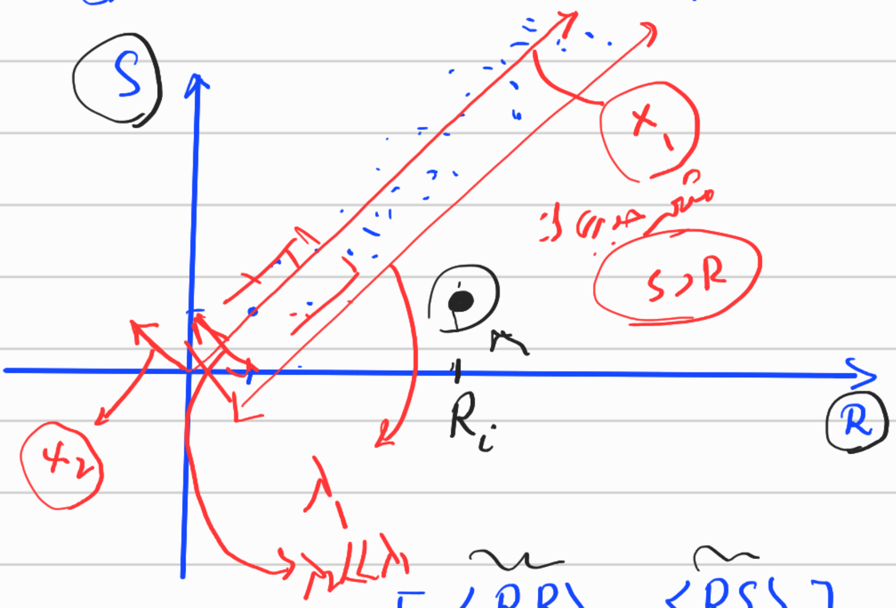


☆ An orthogonal transformation that converts a set of correlated variable to a set of un-correlated variables with dimensional reduction

☆ It is an un-supervised machine learning method

☆ The main goal is to reduce the dimensionality of data set while Preserving the most important information (information loss)

④ An Intuitive example:



R	S
R_1	S_1
R_2	S_2
R_3	S_3
⋮	
R_N	S_N
R_{N+1}	S_{N+1}

$$\underline{\underline{A}} = \mathrm{Cov} = \begin{bmatrix} \langle RR \rangle & \langle RS \rangle \\ \langle SR \rangle & \langle SS \rangle \end{bmatrix}_{2\times 2}$$

$$AX = \lambda X \longrightarrow (A - \lambda 1)X = 0$$

eigen-vector

eigen value

$$|A - \lambda I| = 0 \longrightarrow \underline{\lambda_1, \lambda_2}, X_1, X_2$$

$$A X_1 = \lambda_1 \vec{X_1}$$

$$A X_2 = \lambda_2 \vec{X_2}$$

$$\lambda_1 > \lambda_2$$

$$\updownarrow \qquad \updownarrow$$

اصلی X_1 $\qquad$ X_2 مولفه فرعی

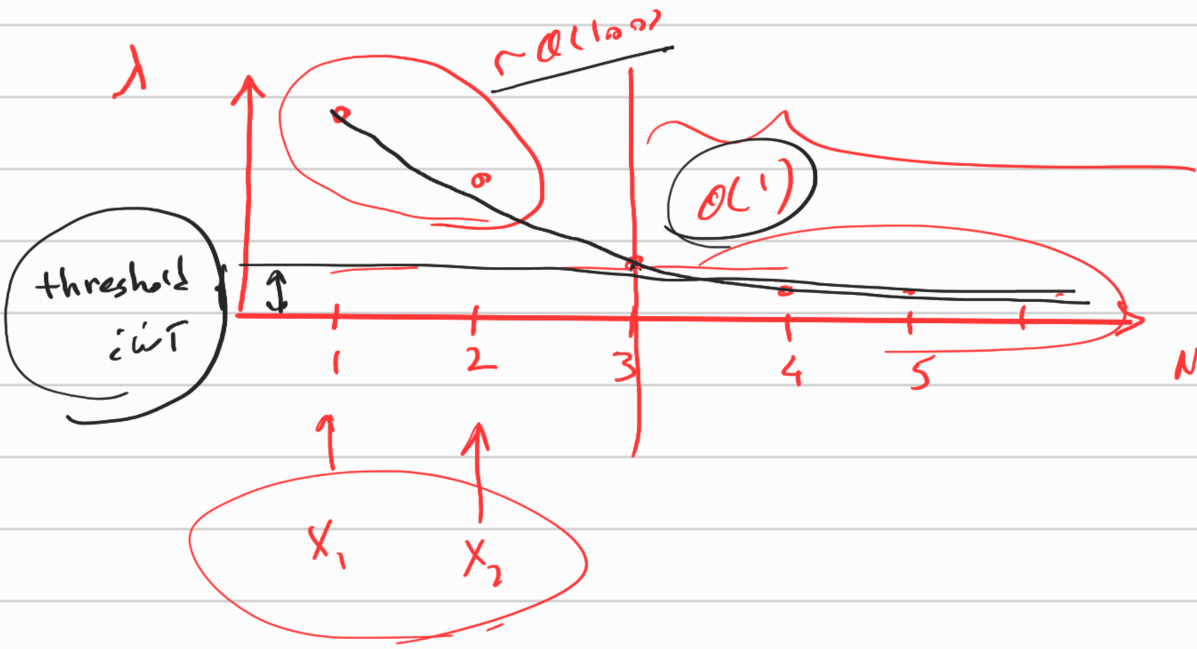
$$X_1 : \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$$

$$X_2 : \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$$

چه کمیت فیزیکی ای ؟ $\leftarrow$ $X_1 = a_1 R + b_1 S$

$$X_2 = a_2 R + b_2 S$$

$\langle X_i | X_j \rangle = \delta_{ij} \longleftarrow$ orthogonal Transformat



⑤ Advantages

- ☆ Dimensionality reduction
- ☆ Feature Selection
- ☆ Noise Reduction $\rightarrow$ Low-Pass Filter

  $\lambda \ll$ حذف

  $\omega \gg$

  $K \ll$
- ☆ Trend Reduction $\rightarrow$ High-Pass Filt

$\rightarrow \lambda$ حذف

$\omega <<$

☆ Outlier Detection داده پرت

⑥ Dis-advantages.

☆ Low interpretability of PCA

☆ Scale-Dependancy. $\longrightarrow$ Z-Transforma

$$\boxed{x \rightarrow x' = \frac{x-\bar{x}}{\sigma}}$$

☆ Linearity.

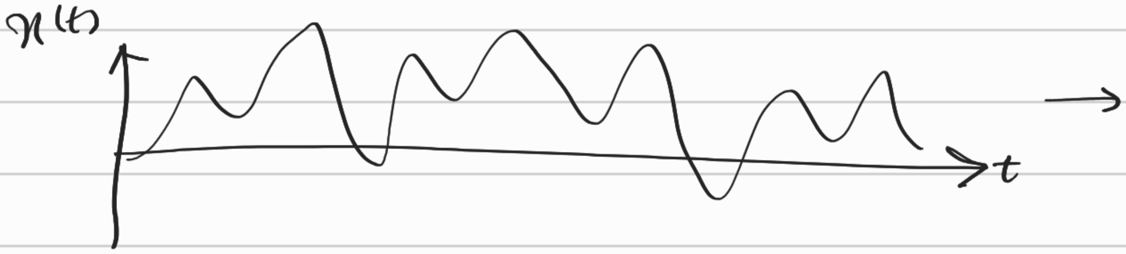
☆ Information Loss

☆ overfitting.

⑦ Singular value Decomposition (SVD)

as a filter

$\{x(t)\} = \{x_1 \ldots \qquad x_N\}$



$\{x(t)\} \longrightarrow$ Matrix ? $\longrightarrow$ Eigen-Value problem

$\lambda_i \qquad |x_i\rangle$