بسم الله الرحمن الرحیم

Making Synthetic Data.

Ex 3: Generating mock data with given $C_x(\tau)$

$\{x_1, x_2, \dots x_N\} \longrightarrow P(x) \sim \mathcal{N}(0,1)$, $C_x(\tau) = \cancel{\langle x(t)\, x(t+\tau)\rangle} = \checkmark$

$P(x) = \mathcal{N}(0,1)$

① $\boxed{C_x(\tau) = \checkmark}$
$\begin{cases} e^{-\tau/T} \\ \sin(\omega\tau) \\ \vdots \end{cases}$ برای مثال

$\begin{cases} P(x) \\ \text{correlat} \end{cases}$ ← ~~تعیین~~ ← هیست داده چیزی است و

Stochastic Data set

② $$S_x(\omega) = \underbrace{\int d\tau\, e^{-i\omega\tau}\, C_x(\tau)}$$

③ Discrete space $T = N\Delta t$

تعداد کل داده‌هایی که می‌خواهیم تولید کنیم

$$\omega_k = \frac{2\pi}{N\Delta t} k \qquad \begin{cases} \omega_{max} = \dfrac{2\pi \cancel{N}}{\Delta t \cancel{N}} \\ \omega_{min} = \dfrac{2\pi}{N\Delta t} \end{cases}$$

④ $$S_x(\omega) \longrightarrow S_x(\omega_k) \longrightarrow S_x(k) = \frac{2\pi}{N\Delta t} S_x(\omega_k)$$

⑤ $$S_x(\omega_k) \text{ or } S_x(k) \xrightarrow{\text{stationarity}} S_x(k) = |X(k)|^2$$

$$X(k) = ? = \sqrt{S_x(k)}\, e^{i\theta_k}$$

تک

$\theta_k \in [0, 2\pi]$

تصادفی

با توجه به اینکه θ_K ها تصادفی هستند

فاز را تصادفی کنیم.

$$= \sqrt{S_x(k)}\,\left[\cos(\theta_k) + i\sin(\theta_k)\right]$$

$$= \sqrt{\frac{S_x(k)}{2}}\,\left[\underbrace{R_1 + i R_2}_{|R|\, e^{i\theta_R}}\right]$$

$$|X(k)|^2 = \frac{S_x(k)}{2}\left[\langle R_1^2\rangle + \langle R_2^2\rangle\right]$$

$$1 \qquad 1$$

⑥

$$x(j) = \frac{1}{N}\sum_{k=1}^{N} X(k)\, e^{+i\overbrace{\frac{2\pi}{N\Delta t}k}^{\omega_k}\,\underbrace{j\Delta t}_{t_j}}$$

$\langle R^2\rangle = 1$, $\langle R\rangle = 0$ تصادفی تولید شد

$$= \frac{1}{N}\sum_{k=1}^{N} \underbrace{\sqrt{\frac{S_x(k)}{2}}\left[R_1(k) + i R_2(k)\right]}_{X(k)}\; e^{\frac{i 2\pi k j}{N}}$$

$$x(j) = \frac{1}{N}\left\{ \sqrt{\frac{S_x(1)}{2}}\left(R_1(1) + i R_2(1)\right] e^{\frac{i 2\pi j}{N}} + \sqrt{\frac{S_x(2)}{2}}\left(R_1(2) + i R_2(2)\right) e^{\frac{i 2\pi 2 j}{N}} \right.$$

$j = 1, N$

$$+ \cdots + \sqrt{\frac{S_x(N)}{2}}\left(R_1(N) + iR_2(N)\right)e^{i2\pi j} \Big\}$$

$$x(j) = \xrightarrow{\text{Z-Transformation}} \quad \langle x(j) \rangle_j = 0$$

$$\langle x^2(j) \rangle_j = 1$$