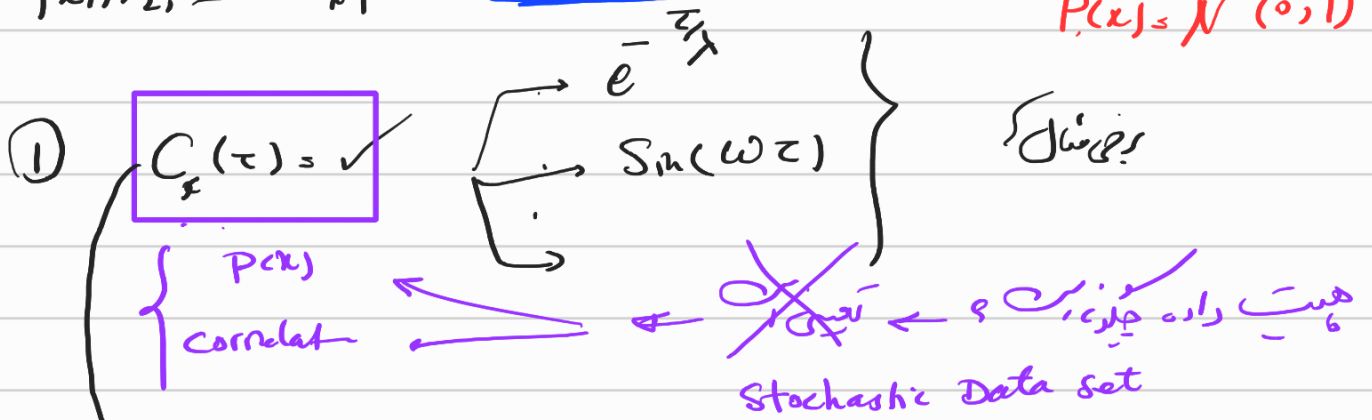


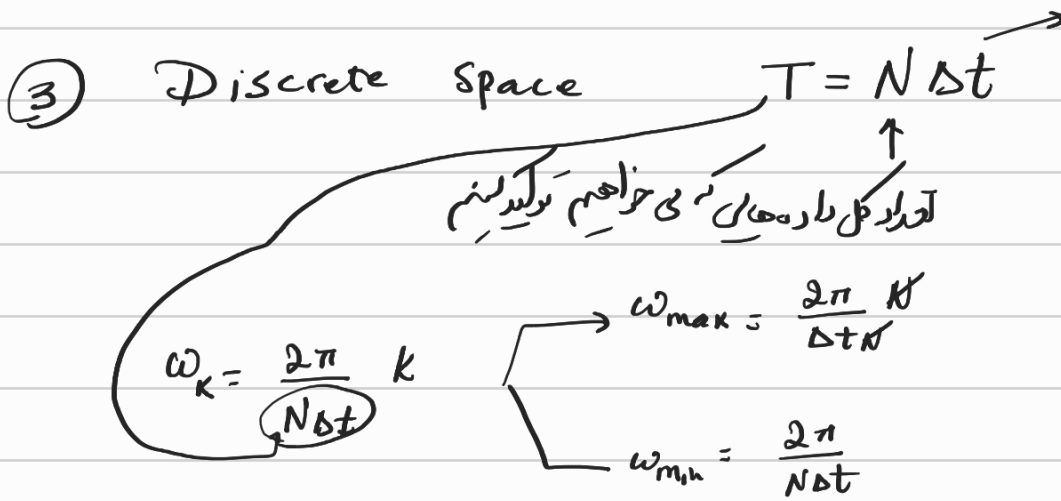
## Making Synthetic Data.

Ex 3: Generating mock data with given  $C_x(\tau)$

$\{x_1, x_2, \dots, x_n\} \rightarrow P(x) \sim \mathcal{N}(0, 1)$ ,  $C_x(\tau) = \langle x(t) x(t+\tau) \rangle = \checkmark$   
 $P(x) = \mathcal{N}(0, 1)$



②  $S_x(\omega) = \int d\tau e^{-i\omega\tau} C_x(\tau)$



④  $S_x(\omega) \rightarrow S_x(\omega_k) \rightarrow S_x(k) = \frac{2\pi}{N\Delta t} S_x(\omega_k)$

⑤  $S_x(\omega_k)$  or  $S_x(k)$   $\xrightarrow{\text{stationarity}}$   $S_x(k) = |X(k)|^2$

$\uparrow$

$$X(k) = ? = \sqrt{S_x(k)} e^{i\theta_k}$$

$\theta_k \in [0, 2\pi]$   
 (فاز) (صاف)

با توجه به اینکه  $\theta_k$  که فاز در حقیقت  
 فاز را صاف می کند

$$= \sqrt{S_x(k)} [\cos(\theta_k) + i\sin(\theta_k)]$$

$$= \sqrt{\frac{S_x(k)}{2}} [R_1 + iR_2]$$

$|R| e^{i\theta_R}$

$$|X(k)|^2 = \frac{S_x(k)}{2} [R_1^2 + R_2^2]$$

$\uparrow$        $\uparrow$   
 1      1

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$$x(j) = \frac{1}{N} \sum_{k=1}^N X(k) e^{+i \frac{2\pi k}{N} j}$$

$\omega_k$   
 Not  $t_j$

$\langle R_1^2 \rangle, \langle R_2^2 \rangle = \dots$

$$= \frac{1}{N} \sum_{k=1}^N \underbrace{\sqrt{\frac{S_x(k)}{2}} [R_1(k) + iR_2(k)]}_{X(k)} e^{i \frac{2\pi k j}{N}}$$

$x(j)$   
 $j=1, N$

$$= \frac{1}{N} \left[ \sqrt{\frac{S_x(1)}{2}} (R_1(1) + iR_2(1)) e^{i \frac{2\pi j}{N}} + \sqrt{\frac{S_x(2)}{2}} (R_1(2) + iR_2(2)) e^{i \frac{2\pi 2j}{N}} \right]$$

$$+ \dots + \left. \sqrt{\frac{S_2(\omega)}{2}} (R_1(\omega) + i R_2(\omega)) e^{i2n\omega} \right\}$$

$$X(j\omega) \xrightarrow{\text{Z-Transformation}}$$

$$\langle x(j\omega) \rangle_j = 0$$

$$\langle x^2(j\omega) \rangle_j = 1$$