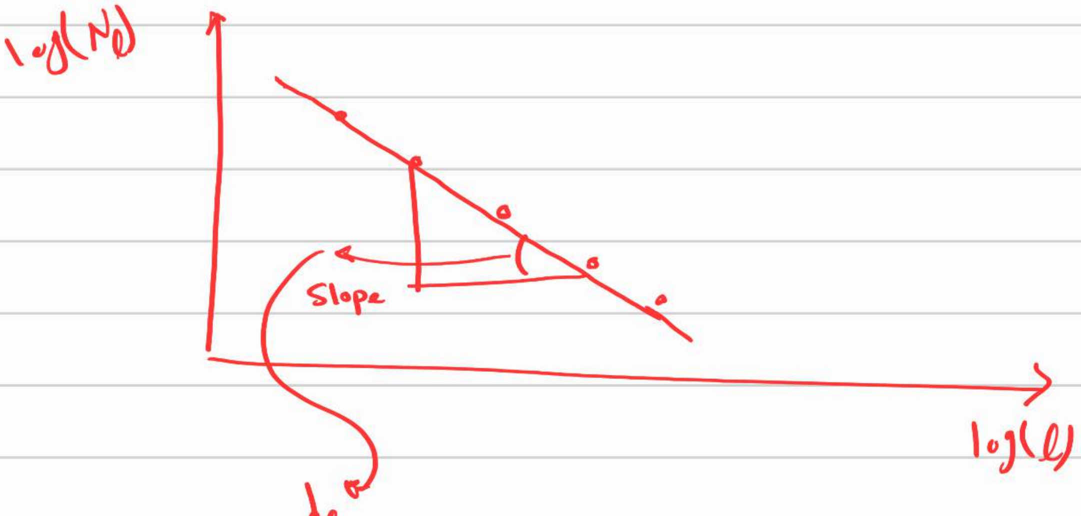
$l = 1/2$ $\quad 3 \sim \left(\frac{1}{2}\right)^{-d_f}$ $\quad \rightarrow \quad \log 3 = d_f \log(2)$ $\quad d_f = \frac{\log 3}{\log 2}$

$l = 1/4$ $\quad 9 \sim (1/4)^{-d_f'}$ $\quad \log 9 = d_f \log(4)$ $\quad d_f = \frac{\log 9}{\log 4} = \frac{\ln 3}{\ln 2}$

$l = \frac{1}{8}$ $\quad N_{1/8} \sim (1/8)^{-d_f''}$

$l = \frac{1}{16}$ $\quad N_{1/16} \sim \left(\frac{1}{16}\right)^{-d_f'''}$



$N(l) = l^{-d_f}$

بسم الله الرحمن الرحیم

Data Analysis تحلیل داده

1: PDF , one-point Statistics , two-point . . .

$P_\xi(x) = \checkmark$ $\qquad P(x_1, t_1 ; x_2, t_2)$ . . .

$$\Downarrow$$

$\boxed{\text{Correlation}}$

2: Correlation

3: Transformation $\longrightarrow$ $\underbrace{\text{Fourier Transformation}}_{e^{i\omega t}}$

3.1 Power Spectrum

$$S_x(\omega) = \int d\tau \, e^{-i\omega\tau} \underbrace{C_x(\tau)}$$

Stationarity assumption

فرض ثابتی

$$S_x(\omega) = \underbrace{\langle X(\omega) X(\omega') \rangle}_{= |X(\omega)|^2} \delta_D(\omega + \omega') 2\pi$$

Bi Spectrum $\langle X(\omega_1) X(\omega_2) X(\omega_3) \rangle$

Trispectrum $\langle X(\omega_1) X(\omega_2) X(\omega_3) X(\omega_4) \rangle$

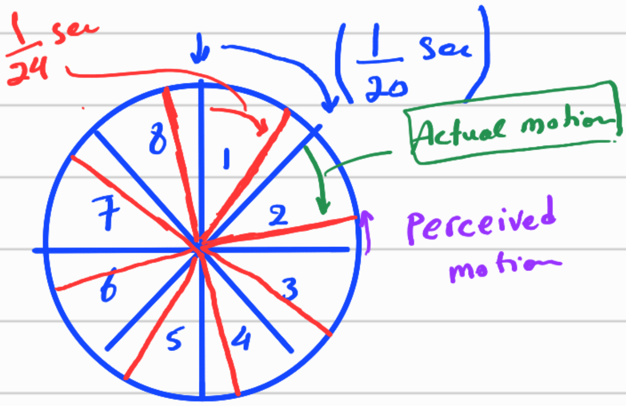
$\Rightarrow$ Non-Gaussianity Test

3.2 Nyquist phenomenon

$f_N \geqslant 2 f_s$

$f_s \leqslant f_{N/2}$

Wagon wheel effect.



$$\begin{cases} f_m = 180 \text{ rpm} \\ f_s = \frac{180}{60} \text{ rps} = 3 \text{ rps} \\ f_p = 3 \times 8 = 24 \text{ frame/s} \end{cases} \quad — \quad f_N \geqslant 48 \text{ frame/s}$$

فرض کنیم

$f_p = 20$ frame/sec $\longrightarrow$ $f_N \geqslant 40$ frame/s

$f_N = 24$ frame/sec

3.3. Surrogate and shuffled

Surrogate $\rightarrow$ تابع توزیع را بررسی می کند

shuffled $\leftarrow$ همبستگی را حذف می کند

3.4. Time sampling and frequency Power Spectrum Analysis

Discrete Series

$$\{x(t_i)\} : \{x_1, x_2, \ldots \quad x_N\}.$$

$$\begin{array}{cccc} x_1 & x_2 & \cdots & x_N \\ \downarrow & \downarrow & & \downarrow \\ t_1 & t_2 & & t_N \end{array}$$

$$t_{i+1} - t_i = \Delta t$$

دارای یک بعد زمان است

Dimension

$$t_{min} = \Delta t$$

$$t_j = j\Delta t$$

$$t_{max} = N\Delta t = t_N = T$$

$$\omega_{min} = \frac{2\pi}{T} = \frac{2\pi}{N\Delta t}$$

$$f_{min} = \frac{1}{N\Delta t}$$

$$\omega_{max} = \frac{2\pi}{\Delta t} \longrightarrow f_{max} = \frac{1}{\Delta t} = \frac{\omega_{max}}{2\pi}$$

$$\omega_k = \frac{2\pi}{N\Delta t} k \qquad k = 1, \ldots N$$

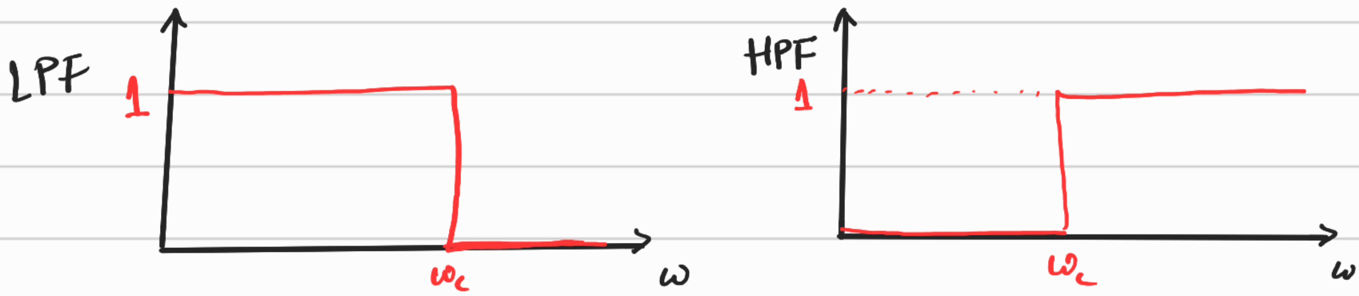
$$\rightarrow f_k = \frac{k}{N\Delta t} \qquad k = 1, \ldots N$$

$$x(t) \longrightarrow x_j = \sum_{k=0}^{N-1} e^{i \frac{2\pi}{N\Delta t} k \Delta t j} X_k$$

$$= \int d\omega \, e^{i\omega t} X(\omega)$$

$$\Delta t = 1\,\text{Sec} \qquad f_k = \frac{k}{N\Delta t} = \frac{k}{T}\ [Hz]$$

3.5: Low-pass filter & High-pass filter



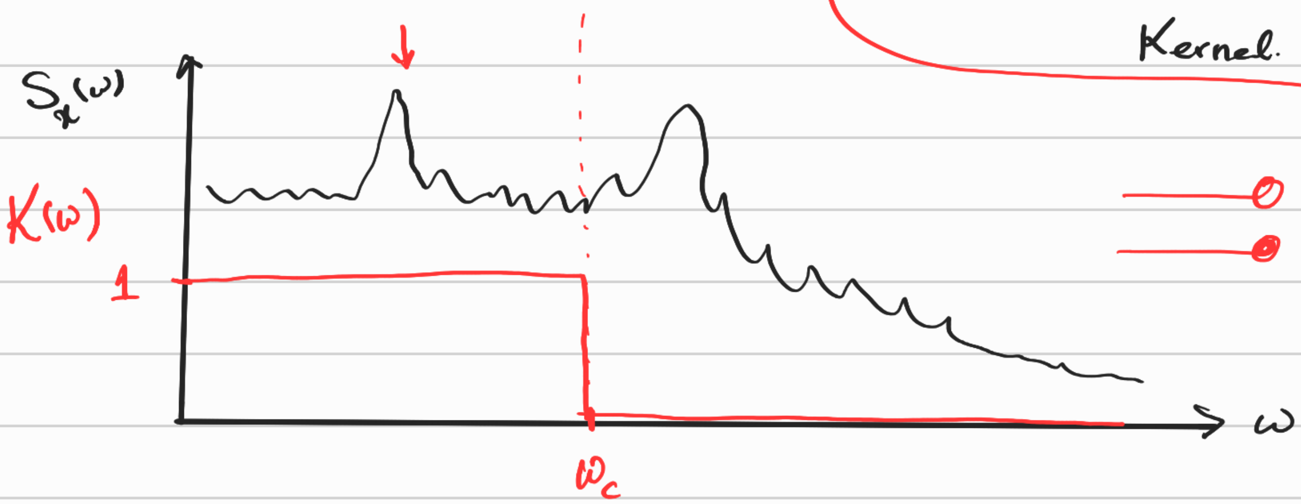
$\underbrace{\{x(t)\};\ \{x_1 \ldots x_N\}.}$

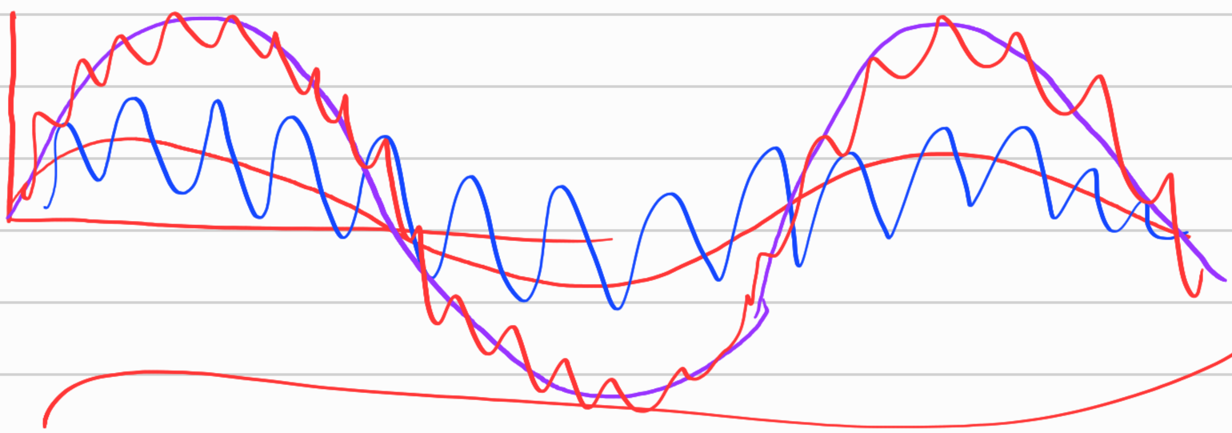
$$S_x(\omega) = \int d\tau\, e^{-i\omega\tau}\, C_x(\tau)$$

$$= |X(\omega)|^2 = \left| \int dt\, e^{-i\omega t} x(t) \right|^2$$

$$S_x(\omega) \xrightarrow{\text{LPF}} \tilde{S}_x(\omega) = \int d\omega' \underbrace{K(\omega, \omega')}\, S_x(\omega')$$

Kernel.





$$K = \Theta(\omega_c - \omega) = \begin{cases} 1 & \omega \leq \omega_c \\ 0 & \omega > \omega_c \end{cases}$$

Kernel

LPF — $$\tilde{S}(\omega) = \begin{cases} S(\omega) & \omega \leq \omega_c \quad K \leq K_c \\ 0 & \omega > \omega_c \quad K > K_c \end{cases}$$

$K_c = ?$

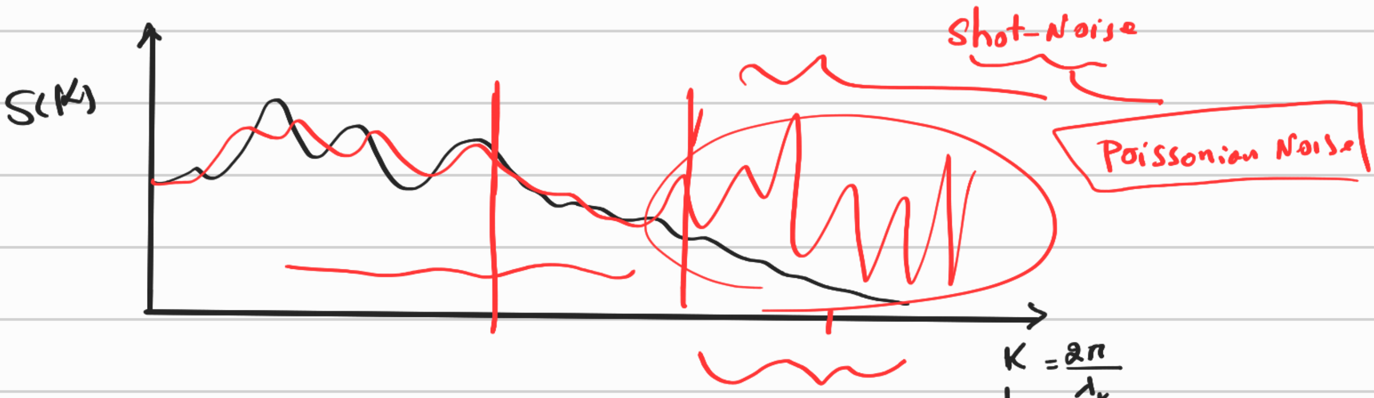
$$\omega_K = \frac{2\pi}{N\Delta t} K$$

$$\omega_c = \frac{2\pi}{N\Delta t} K_c \rightarrow K_c = \left[\frac{\omega_c N \Delta t}{2\pi}\right]$$

HPF — $$\tilde{S}(\omega) = \begin{cases} 0 & \omega \leq \omega_c \quad K \leq K_c \\ S(\omega) & \omega > \omega_c \quad K > K_c \end{cases}$$

3.5-1 ☆ Trend — Detrending روند

☆ Noise → Denoising نوفه



$K = \frac{2\pi}{\lambda_K}$

↓

عدد موج

$K = \sqrt{K_x^2 + K_y^2}$

Denoising

نقص نامطلوب

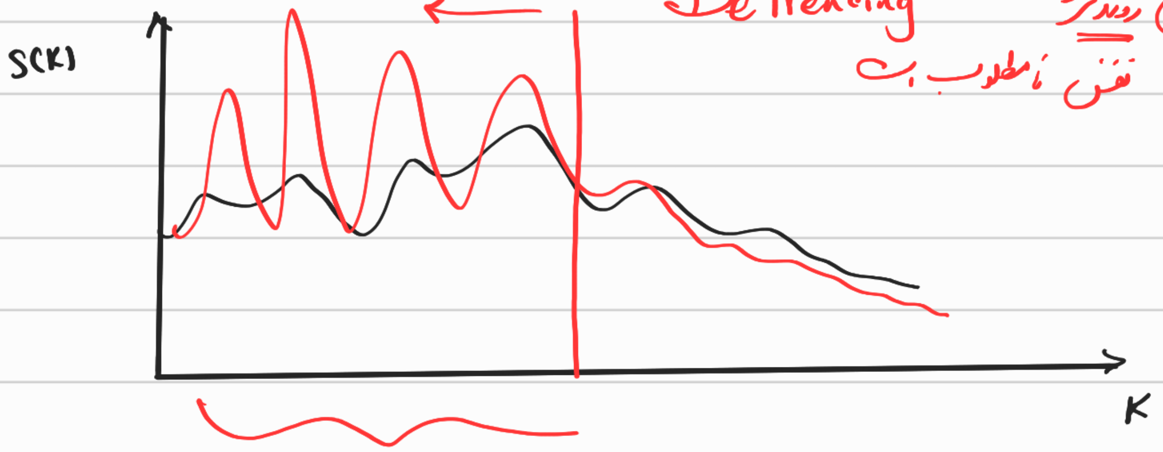
$$K_x = \frac{2\pi}{L} n_x \qquad n_x = 1 \ldots N_{pix}^{x} \qquad N_{pix} = N_{pix}^{x} N_{pix}^{y}$$

$$K_y = \frac{2\pi}{L} n_y \qquad n_y = 1 \ldots N_{pix}^{y}$$

حذف

Detrending — بدون روند کردن

نقش نامطلوب ب



☆ شکل نشان خوام داد ☆

Data generator

SVD ←

Imf ←

Wavelet *

بسم الله الرحمن الرحیم

Mock Data generation
Synthetic

تولید داده‌های تصنعی

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① Motivations → Ⓐ Ensemble average
متوسط‌گیری هنگردی

$\langle f \rangle_{ens}$

Ⓑ Simulation — شبیه‌سازی
↓
observable collection
جمع آوری مشاهده‌پذیرها.

Ⓒ Forecast — پیش‌یابی

② Data generation
→ Ⓐ Effective approach — رهیافت مؤثر
→ Ⓑ Simulation — شبیه‌سازی

Notes: Effective Theory — نظریه مؤثر

Ex1: Langevin Equation $\dot{v} = -\gamma v + \underset{\sim}{\eta(t)}$

Ex 2: Landau – Ginzburg – Wilson theory.

For a Magnetization system (Ferromagnetism)

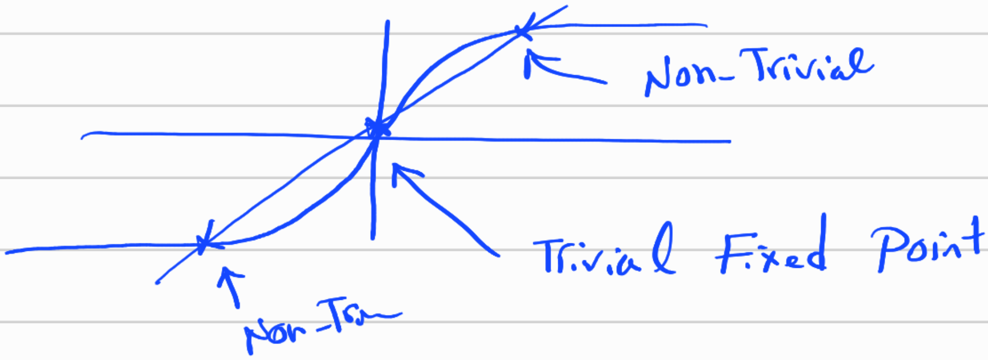
$$\mathcal{H} = -J \sum \vec{m}_i \cdot \vec{m}_j - \vec{B}_{ext} \cdot \sum \vec{m}_i$$

$J > 0 \longrightarrow$ Ferromagnetism $\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow$

$J < 0 \longrightarrow$ Anti Ferromagnetism $\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow$

$B_{ext} = 0$

$$M = \left|\sum \vec{m}_i\right| = \tanh(\alpha M)$$



$$\mathcal{H} = \int d^3 r \left[\underbrace{a_1 (\vec{\nabla} m)^2 + a_2 m^2 + a_3 m^3 + a_4 m^4 - B_{ext} m \cdots}_{} \right]$$

$$Z = \mathrm{Tr}\left(e^{-\beta \mathcal{H}}\right) = \int \frac{d^N q \, d^N p}{(2\pi)^N} e^{-\beta \mathcal{H}} = \int d\Gamma e^{-\beta \mathcal{H}(\Gamma)}$$

$$= \sum_{\{m_i\}} e^{-\beta \mathcal{H}([m])}$$

$$F = -K_B T \ln Z$$ پتانسیل ترمودینامیکی

$\downarrow$

Helmholtz free Energy

$$M = \frac{\partial F}{\partial B_{ext}}\bigg|_{B_{ext}}$$

$$\chi = \frac{\partial M}{\partial B_{ext}}\bigg|_{B=0}$$

$\vdots$

③ Ex1: Computer Random Generator

R = Call Random Number — Vanilla model.

Some Properties

$$P(R(t)) = cts \qquad \{R(t_1), R(t_2), R(t_3), \dots R(t_N)\}$$

سری مشاهده پذیر نوعی

$$R \in (0,1)$$

$$C_R(\tau) = \langle R(t) R(t+\tau) \rangle = \delta_D(\tau)$$

Weighted TPCF

Stationarity نا ایستا ~~Static~~

Ex2: Data generation with given PDF

☆ Box-Muller

☆ PDF-Transformation $\{\xi\}$ — $P(\xi)$ ✓

$\{\eta\}$ — $P(\eta)$ ✓

?

$\eta \stackrel{?}{=} g(\xi) = ?$

$$\int_{-\infty}^{\xi} d\xi' \, P(\xi') = \int_{-\infty}^{\eta} d\eta' \, P(\eta')$$

☆ Von-Neumann method

Metro-polis "

$$C_\eta(\tau) = \langle \eta(t)\eta(t+\tau) \rangle \propto \delta_D(\tau)$$

$$P(\eta(t)) = \text{given}$$

Ex 3: